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A Stochastic Thin-Layer Method for an Inhomogeneous Half-space in Antiplane Shear

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Abstract

A stochastic thin-layer method is developed for the analysis of wave propagation in an inhomogeneous half-space in antiplane shear. The shear modulus is assumed uncertain and characterized by a random field. It is expanded by the polynomial chaos expansion and discretized by the Karhunen-Loève expansion. The half-space is represented by thin layers which include not only ordinary layers but also continued-fraction absorbing boundary conditions for its infinite extent. Applying the Galerkin method both in the spatial and stochastic domains, a stochastic thin-layer method for an inhomogeneous half-space in antiplane shear is presented. The developed stochastic methods are found to provide accurate probabilistic treatment of half-space dynamics.

Keywords: stochastic analysis, thin-layer method, inhomogeneous half-space, uncertainty, soil-structure interaction, wave propagation.

1 Introduction

Wave propagation in a layered half-space has many applications in seismology and civil engineering. Formulations based on transfer matrices were presented for wave propagation in layered media by Thomson [1] and Haskell [2]. Kausel and Roësset [3] presented exact stiffness matrices for a layered half-space. Also, discrete stiffness matrices which are approximations of the exact ones were given by Kausel and Roësset [3]. Subdividing a finite layer into several thin layers and adopting simple interpolation functions in the direction of layering, discrete layer stiffness matrices can be obtained. The method based on the discrete stiffness matrices is often referred to as a 'thin-layer method'. Kausel [4] developed a spectral decomposition for solutions to the thin-layer method. Since the method leads to rigorous and effective numerical models, it has been applied to wave-propagation problems in various layered systems [5]. In the conventional thin-layer method, properties of layered media have been assumed deterministic. However, all natural and man-made systems have intrinsic randomness in material properties. Therefore, a stochastic enhancement of the thin-layer method is highly desirable.

For stochastic seismic analysis of soil deposits, the thin-layer method and consistent transmitting boundary combined with Monte Carlo simulations were applied to the seismic analysis of heterogeneous soil [6, 7]. Assuming vertically propagating seismic waves, perturbation approach and spectral stochastic finite-element method were developed in the framework of the thin-layer method [8]. However, effects of the infinite half-space were not considered rigorously in these studies.

In this study, a "stochastic thin-layer method" is developed for analysis of wave propagation in an inhomogeneous half-space in antiplane shear. The shear modulus is assumed uncertain and characterized by a random field with vertically varying statistical properties. An infinite extent of the half-space is represented by continued-fraction absorbing boundary conditions (CFABCs) [9, 10]. Considering the random field and including the CFABCs, a stochastic thin-layer method in an inhomogeneous half-space in antiplane shear is developed.

2 Formulation of a stochastic thin-layer method

2.1 Karhunen-Loève expansion of a random shear modulus

An inhomogeneous half-space with random shear modulus is considered. The random shear modulus is characterized by a random field and has vertically varying statistical properties. Mean and standard deviation of the shear modulus is denoted by $\mu(z)$ and $\sigma(z)$, respectively. Since the shear modulus has only positive values, it is assumed to have a log-normal distribution. Then, it can be expanded by the polynomial chaos expansion [11]:

$$G(z) = \exp[\lambda(z) + \zeta(z)\xi(z)] = \mu(z)\sum_{p=0}^{\infty}\frac{[\zeta(z)]^p}{p!}\Gamma_p^{(1)}[\xi(z)] \approx \mu(z)\sum_{p=0}^{N_G}\frac{[\zeta(z)]^p}{p!}\Gamma_p^{(1)}[\xi(z)] \tag{1a}$$

$$\lambda(z) = \ln \mu(z) - \frac{1}{2}\ln\left(1 + \left[\frac{\sigma(z)}{\mu(z)}\right]^2\right) \tag{1b}$$

$$\zeta(z) = \ln\left(1 + \left[\frac{\sigma(z)}{\mu(z)}\right]^2\right) \tag{1c}$$

where $\lambda(z)$ and $\zeta(z)$ are mean and standard deviation of the natural logarithm of the shear modulus, respectively. $\Gamma_p^{(1)}[\xi(z)]$ in Equation (1a) is the one-dimensional pth-order Hermite polynomial chaos of a zero-mean and unit-variance Gaussian field $\xi(z)$. The field $\xi(z)$ has a correlation function $C(z_1, z_2)$. Using a Karhunen-

Loève expansion, the Gaussian field $\xi(z)$ is represented by independent standard normal random variables ξ_i's [12]:

$$\xi(z)=\sum_{i=1}^{\infty}\sqrt{\lambda_i}f_i(z)\xi_i \approx \sum_{i=1}^{s}\sqrt{\lambda_i}f_i(z)\xi_i = \sum_{i=1}^{s}a_i(z)\xi_i \tag{2a}$$

where λ_i and $f_i(z)$ are eigenvalue and eigenfunction of the correlation function $C(z_1,z_2)$. The correlation function $C(z_1,z_2)$ is approximated as follows:

$$C(z_1,z_2)=\sum_{i=1}^{\infty}\lambda_i f_i(z_1)f_i(z_2)\approx\sum_{i=1}^{s}\lambda_i f_i(z_1)f_i(z_2) \tag{2b}$$

Since $\xi(z)$ has a unit-variance, $\sum_{i=1}^{s}\left[\sqrt{\lambda_i}f_i(z)\right]^2=\sum_{i=1}^{s}\left[a_i(z)\right]^2=1$. Then, $\Gamma_p^{(1)}\left[\xi(z)\right]$ can be expressed as follows [13]:

$$\begin{aligned}\Gamma_p^{(1)}\left[\xi(z)\right]&=\Gamma_p^{(1)}\left[\sum_{i=1}^{s}a_i(z)\xi_i\right]\\&=\sum_{p_1+\cdots+p_s=p}\frac{p!}{p_1!\cdots p_s!}\left[a_1(z)\right]^{p_1}\cdots\left[a_r(z)\right]^{p_s}\Gamma_{p_1}^{(1)}(\xi_1)\cdots\Gamma_{p_s}^{(1)}(\xi_s)\\&=\sum_{p_1+\cdots+p_s=p}\frac{p!}{p_1!\cdots p_s!}\left[a_1(z)\right]^{p_1}\cdots\left[a_r(z)\right]^{p_s}\Gamma_{p_1\cdots p_s}^{(r)}(\xi_1,\cdots,\xi_s)\end{aligned} \tag{3}$$

where $\Gamma_{p_1\cdots p_s}^{(s)}(\xi_1,\cdots,\xi_s)=\Gamma_{p_1}^{(1)}(\xi_1)\cdots\Gamma_{p_s}^{(1)}(\xi_s)$ is the s-dimensional pth-order Hermite polynomial chaos of independent standard normal random variables ξ_i's. Finally, the random shear modulus in an inhomogeneous half-space can be expanded as follows:

$$\begin{aligned}G(z)&=\exp\left[\lambda(z)+\zeta(z)\xi(z)\right]=\mu(z)\sum_{p=0}^{\infty}\frac{\left[\zeta(z)\right]^p}{p!}\Gamma_p^{(1)}\left[\xi(z)\right]\\&\approx\mu(z)\sum_{p=0}^{N_G}\frac{\left[\zeta(z)\right]^p}{p!}\Gamma_p^{(1)}\left[\xi(z)\right]\\&=\mu(z)\sum_{p=0}^{N_G}\sum_{p_1+\cdots+p_s=p}\frac{\left[\zeta(z)\right]^p}{p_1!\cdots p_s!}\left[a_1(z)\right]^{p_1}\cdots\left[a_s(z)\right]^{p_s}\Gamma_{p_1\cdots p_s}^{(s)}(\xi_1,\cdots,\xi_s)\end{aligned} \tag{4}$$

2.2 Stochastic thin-layer method for an inhomogeneous half-space

A stochastic thin-layer method for an inhomogeneous half-space in antiplane shear is formulated. The half-space is represented by N thin layers (Figure 1). The layers include not only ordinary layers but also continued-fraction absorbing boundary conditions for the infinite extent of the half-space. Since each ordinary layer is thin enough, the shear modulus in each layer is assumed constant. Fixity is assumed at the base of the layered system.

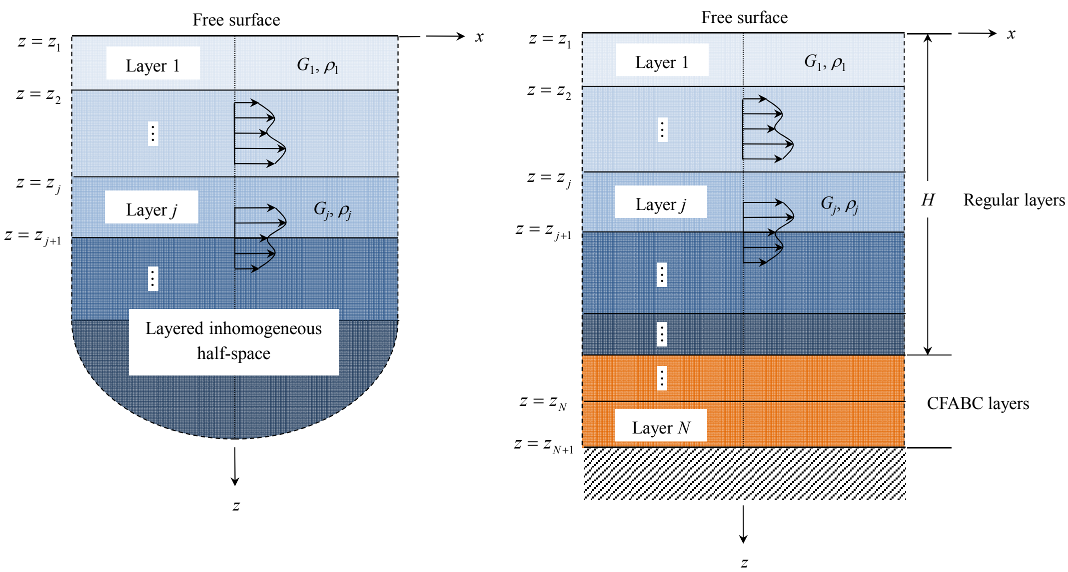
In layer j, the governing differential equation in antiplane shear is given as follows [14, 15]:

$$G_j \frac{\partial^2 v}{\partial x^2} + G_j \frac{\partial^2 v}{\partial z^2} - \rho_j \frac{\partial^2 v}{\partial t^2} + p_y = 0 \tag{5}$$

where $v(x,z,t)$ is a transverse displacement and $p_y(x,z,t)$ is a transverse body force. In Equation (5), G_j and ρ_j are the shear modulus and density of the layer j, respectively. G_j is determined from Equation (4):

$$\begin{aligned} G_j = G(Z_j) &= \mu(Z_j) \sum_{p=0}^{N_G} \sum_{p_1+\cdots+p_s=p} \frac{[\zeta(Z_j)]^p}{p_1! \cdots p_s!} [a_1(Z_j)]^{p_1} \cdots [a_s(Z_j)]^{p_s} \Gamma^{(s)}_{p_1 \cdots p_s}(\xi_1, \cdots, \xi_s) \\ &= \mu_j \sum_{p=0}^{N_G} \sum_{p_1+\cdots+p_s=p} \frac{\zeta_j^p}{p_1! \cdots p_s!} a_{1,j}^{p_1} \cdots a_{s,j}^{p_s} \Gamma^{(s)}_{p_1 \cdots p_s}(\xi_1, \cdots, \xi_s) \end{aligned} \tag{6}$$

where $z_j \le Z_j \le z_{j+1}$ for ordinary layers and $Z_j = H$ for CFABC layers.



(a) Schematic view (b) Thin layers

Figure 1: Layered inhomogeneous half-space.

The displacement and the body force are assumed *x*-harmonic and time-harmonic. Since the shear moduli are random variables, the displacement in Eqs. (5) is also a random variable. Then, the random displacement and the deterministic body force can be expressed as follows:

$$v(x,z,t;\xi_1,\cdots,\xi_s)=V(z;\xi_1,\cdots,\xi_s)e^{-ikx}e^{i\omega t}=\sum_{q=0}^{\infty}\sum_{q_1+\cdots+q_s=q}V_{q_1\cdots q_s}(z)\Gamma^{(s)}_{q_1\cdots q_s}(\xi_1,\cdots,\xi_s)e^{-ikx}e^{i\omega t}$$
$$\approx\sum_{q=0}^{N_d}\sum_{q_1+\cdots+q_s=q}V_{q_1\cdots q_s}(z)\Gamma^{(r)}_{q_1\cdots q_s}(\xi_1,\cdots,\xi_s)e^{-ikx}e^{i\omega t} \tag{7a}$$

$$p_y(x,z,t)=P_y(z)e^{-ikx}e^{i\omega t} \tag{7b}$$

where k is the wavenumber and ω the frequency of excitation. Inserting Eqs. (7) into Eqs. (5), it can be shown that $V_{q_1\cdots q_s}(z)$ must satisfy the following equations:

$$\sum_{q=0}^{N_d}\sum_{q_1+\cdots+q_s=q}\sum_{p=0}^{N_G}\sum_{p_1+\cdots+p_s=p}\frac{\zeta_j^p}{p_1!\cdots p_s!}a_{1,j}^{p_1}\cdots a_{s,j}^{p_s}$$
$$\times\left(-k^2\mu_jV_{q_1\cdots q_s}+\mu_j\frac{d^2V_{q_1\cdots q_s}}{dz^2}+\delta_{p0}\rho_j\omega^2V_{q_1\cdots q_s}\right)\Gamma^{(s)}_{p_1\cdots p_s}\Gamma^{(s)}_{q_1\cdots q_s}+P_y=0 \tag{8}$$

where δ_{p0} is the Kronecker delta.

The Galerkin method can be applied to Equation (8) not only in the spatial domain of *z* but also in the stochastic domains of ξ_i's. Virtual displacement is also expanded by the polynomial chaos expansion in the stochastic domains:

$$\delta V(z;\xi_1,\cdots,\xi_s)=\sum_{r=0}^{\infty}\sum_{r_1+\cdots+r_s=r}\delta V_{r_1\cdots r_s}(z)\Gamma^{(s)}_{r_1\cdots r_s}(\xi_1,\cdots,\xi_s)$$
$$\approx\sum_{r=0}^{N_d}\sum_{r_1+\cdots+r_s=r}\delta V_{r_1\cdots r_s}(z)\Gamma^{(s)}_{r_1\cdots r_s}(\xi_1,\cdots,\xi_s) \tag{9}$$

Interpolating the actual and virtual displacements in the spatial domain, applying the Galerkin method to Equation (8), and assembling equations for all layers with displacement continuity and stress equilibrium between the layers and the fixed boundary condition at the base, a final discrete governing equation can be obtained:

$$\left(k^2\begin{bmatrix}\mathbb{A}_{00} & \mathbb{A}_{01} & \cdots & \mathbb{A}_{0N_d}\\ \mathbb{A}_{10} & \mathbb{A}_{11} & \cdots & \mathbb{A}_{1N_d}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbb{A}_{N_d0} & \mathbb{A}_{N_d1} & \cdots & \mathbb{A}_{N_dN_d}\end{bmatrix}+\begin{bmatrix}\mathbb{G}_{00} & \mathbb{G}_{01} & \cdots & \mathbb{G}_{0N_d}\\ \mathbb{G}_{10} & \mathbb{G}_{11} & \cdots & \mathbb{G}_{1N_d}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbb{G}_{N_d0} & \mathbb{G}_{N_d1} & \cdots & \mathbb{G}_{N_dN_d}\end{bmatrix}\right.$$
$$\left.-\omega^2\begin{bmatrix}\mathbb{M}_{00} & \mathbb{M}_{01} & \cdots & \mathbb{M}_{0N_d}\\ \mathbb{M}_{10} & \mathbb{M}_{11} & \cdots & \mathbb{M}_{1N_d}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbb{M}_{N_d0} & \mathbb{M}_{N_d1} & \cdots & \mathbb{M}_{N_dN_d}\end{bmatrix}\right)\begin{Bmatrix}\mathbb{V}_0\\ \mathbb{V}_1\\ \vdots\\ \mathbb{V}_{N_d}\end{Bmatrix}=\begin{Bmatrix}\mathbb{P}_0\\ \mathbb{P}_1\\ \vdots\\ \mathbb{P}_{N_d}\end{Bmatrix} \tag{10}$$

The matrices $\mathbb{A}_{rq}$, $\mathbb{B}_{rq}$, $\mathbb{G}_{rq}$, and $\mathbb{M}_{rq}$ in Equation (9) have elememt blocks of $\sum_{p=0}^{N_G}\sum_{p_1+\cdots+p_s=p}\varepsilon_{(r_1\cdots r_s)(q_1\cdots q_s)(p_1\cdots p_s)}\mathbf{A}_{p_1\cdots p_s}$, $\sum_{p=0}^{N_G}\sum_{p_1+\cdots+p_s=p}\varepsilon_{(r_1\cdots r_s)(q_1\cdots q_s)(p_1\cdots p_s)}\mathbf{B}_{p_1\cdots p_s}$, $\sum_{p=0}^{N_G}\sum_{p_1+\cdots+p_s=p}\varepsilon_{(r_1\cdots r_s)(q_1\cdots q_s)(p_1\cdots p_s)}\mathbf{G}_{p_1\cdots p_s}$, and $\sum_{p=0}^{N_G}\sum_{p_1+\cdots+p_s=p}\varepsilon_{(r_1\cdots r_s)(q_1\cdots q_s)(p_1\cdots p_s)}\mathbf{M}_{p_1\cdots p_s}$ where $r_1+\cdots+r_s=r$ and $q_1+\cdots+q_s=q$, respectively. The matrices $\mathbf{A}_{p_1\cdots p_s}$, $\mathbf{B}_{p_1\cdots p_s}$, $\mathbf{G}_{p_1\cdots p_s}$, and $\mathbf{M}_{p_1\cdots p_s}$ are assembled from element matrices. For an ordinary layer *j*, they are given as follows:

$$\mathbf{A}^j_{p_1\cdots p_s}=\frac{\zeta_j^p}{p_1!\cdots p_s!}a_{1,j}^{p_1}\cdots a_{s,j}^{p_s}\frac{\mu_j h_j}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix} \tag{11a}$$

$$\mathbf{G}^j_{p_1\cdots p_s}=\frac{\zeta_j^p}{p_1!\cdots p_s!}a_{1,j}^{p_1}\cdots a_{s,j}^{p_s}\frac{\mu_j}{h_j}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} \tag{11b}$$

$$\mathbf{M}^j_{p_1\cdots p_s}=\delta_{p0}\frac{\zeta_j^p}{p_1!\cdots p_s!}a_{1,j}^{p_1}\cdots a_{s,j}^{p_s}\frac{\rho_j h_j}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix} \tag{11c}$$

where h_j is depth of the layer *j*. For a CFABC layer *j*, the element matrices are expressed as:

$$\mathbf{A}^j_{p_1\cdots p_s}=\frac{C_{pe}}{4}\left(\frac{3}{2}\right)^p\frac{\zeta_j^p}{p_1!\cdots p_s!}a_{1,j}^{p_1}\cdots a_{s,j}^{p_s}\frac{\mu_j\sqrt{\mu_j}}{\sqrt{\rho_j}}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix} \tag{12a}$$

$$\mathbf{G}^j_{p_1\cdots p_s}=\frac{1}{C_{pe}}\left(\frac{1}{2}\right)^p\frac{\zeta_j^p}{p_1!\cdots p_s!}a_{1,j}^{p_1}\cdots a_{s,j}^{p_s}\sqrt{\mu_j\rho_j}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} \tag{12b}$$

$$\mathbf{M}^j_{p_1\cdots p_s}=\frac{C_{pe}}{4}\left(\frac{1}{2}\right)^p\frac{\zeta_j^p}{p_1!\cdots p_s!}a_{1,j}^{p_1}\cdots a_{s,j}^{p_s}\sqrt{\mu_j\rho_j}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix} \tag{12c}$$

where $C_{pe} = -\frac{2i}{\omega \cos\theta_j}$ and $\frac{2}{\omega\sqrt{\alpha_j^2 - 1}}$ for propagating and evanescent waves, respectively. $\varepsilon_{(r_1 \cdots r_s)(q_1 \cdots q_s)(p_1 \cdots p_s)} = E\left[\Gamma^{(s)}_{r_1 \cdots r_s}(\xi_1, \cdots, \xi_s)\Gamma^{(s)}_{q_1 \cdots q_s}(\xi_1, \cdots, \xi_s)\Gamma^{(s)}_{p_1 \cdots p_s}(\xi_1, \cdots, \xi_s)\right]$ $= E\left[\Gamma^{(1)}_{r_1}(\xi_1)\Gamma^{(1)}_{q_1}(\xi_1)\Gamma^{(1)}_{p_1}(\xi_1)\right] \cdots E\left[\Gamma^{(1)}_{r_s}(\xi_s)\Gamma^{(1)}_{q_s}(\xi_s)\Gamma^{(1)}_{p_s}(\xi_s)\right]$. The vectors $\mathbb{V}_q$ and $\mathbb{P}_r$ in Equation (10) have element blocks of $\mathbf{V}_{q_1 \cdots q_s}$ and $\mathbf{P}_{r_1 \cdots r_s}$ where $r_1 + \cdots + r_s = r$ and $q_1 + \cdots + q_s = q$, respectively. $\mathbf{V}_{q_1 \cdots q_s} = \left[V_{q_1 \cdots q_s}(z_1) \ \cdots \ V_{q_1 \cdots q_s}(z_N)\right]^T$, $\mathbf{P}_{r_1 \cdots r_s} = E\left[\Gamma^{(s)}_{r_1 \cdots r_s}(\xi_1, \cdots, \xi_s)\mathbf{P}\right]$ where $\mathbf{P}$ is a vector of nodal forces equivalent to the body force $P_y(z)$. In the definition of $\mathbf{P}_{r_1 \cdots r_s}$ and $\varepsilon_{(r_1 \cdots r_s)(q_1 \cdots q_s)(p_1 \cdots p_s)}$, the operator $E[\cdot]$ is the expectation of a function of the random variables ξ_i's. Since the body force is deterministic, $\mathbb{P}_r = \mathbf{0}$ for $r = 1, \cdots, N_d$.

3 Application

Stochastic dynamic responses of an inhomogeneous half-space with uncertain shear modulus subjected to line loads on its surface (Figure 2) are examined. The random shear modulus is assumed to have a mean $\mu(z) = \mu_0 + \mu_0\left(1 - e^{-3z}\right)$, a standard deviation $\sigma(z) = 0.05\mu(z)$, and a correlation function $C(z_1, z_2) = \exp\left[-|z_1 - z_2| / L_c\right]$ where L_c is a correlation length. Then, the eigenvalue and eigenvector of $C(z_1, z_2)$ are given as follows [12]:

$$\lambda_i = \frac{2L_c}{1 + (\omega_i L_c)^2} \tag{13}$$

$$f_i(z) = \frac{\cos\omega_i\left(z - \frac{H}{2}\right)}{\sqrt{\frac{H}{2} + \frac{\sin(\omega_i H)}{2\omega_i}}} \quad \text{for odd } i \tag{14a}$$

$$f_i(z) = \frac{\sin\omega_i\left(z - \frac{H}{2}\right)}{\sqrt{\frac{H}{2} - \frac{\sin(\omega_i H)}{2\omega_i}}} \quad \text{for even } i \tag{14b}$$

where ω_i is a solution to the transcendental equations

$$\frac{1}{L_c} - \omega_i \tan\left(\frac{\omega_i H}{2}\right) = 0 \quad \text{for odd } i \tag{15a}$$

$$\omega_i + \frac{1}{L_c}\tan\left(\frac{\omega_i H}{2}\right) = 0 \text{ for even } i \tag{15b}$$

It is assumed that $L_c = 6H$. Then, $\omega_i H$, $i = 1,\cdots,6$, are 0.5695, 3.2442, 6.3358, 9.4600, 12.5928, and 15.7292, respectively. The eigenvalues λ_i / H, $i = 1,\cdots,6$, are 0.9468, 0.0316, 0.0083, 0.0037, 0.0021, and 0.0013, respectively. The eigenfunctions $\sqrt{H} f_i(z/H)$, $i = 1,\cdots,6$, are shown in Figure 3. The exact correlation function $C(z_1, z_2)$ and its approximation of Equation (2b) are compared in Figure 4. The random shear modulus is expanded using the polynomial chaos expansion in Equation (4). In the expansion, six independent normal random variables ξ_i, $i = 1,\cdots,6$, are considered, i.e. $s = 6$ and 6-dimensional Hermite polynomial chaos $\Gamma^{(6)}_{p_1 \cdots p_6}(\xi_1,\cdots,\xi_6)$ are used in Equation (4). The expansion is truncated at the 2nd order term, i.e. $N_G = 2$ in Equation (4).

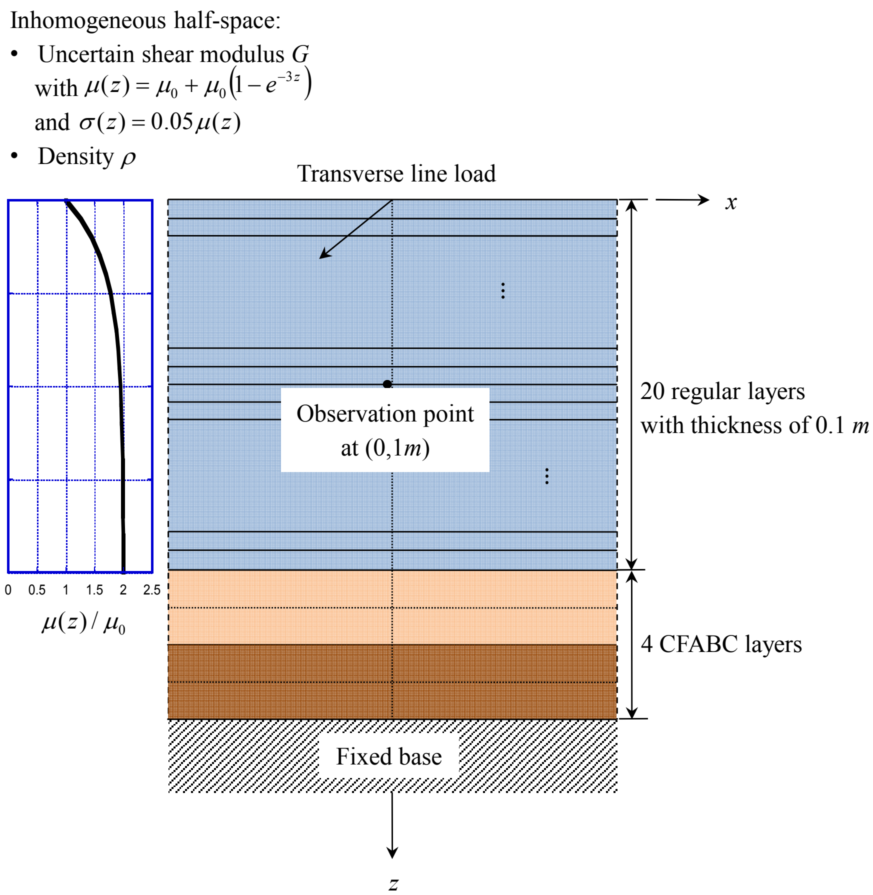


Figure 2: Application.

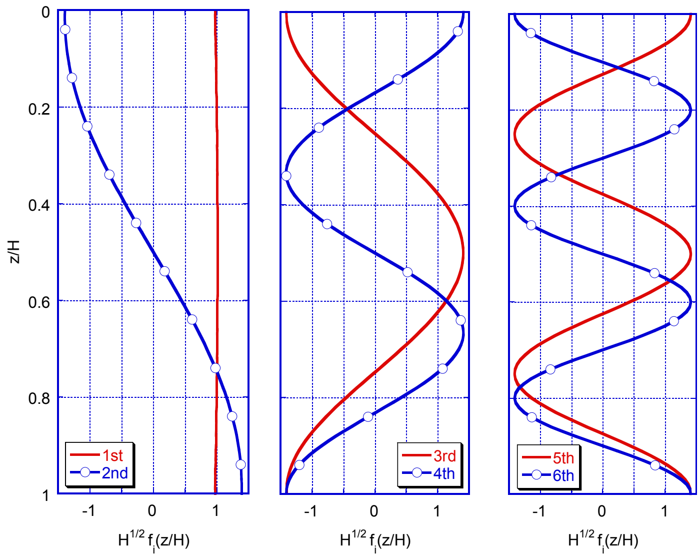
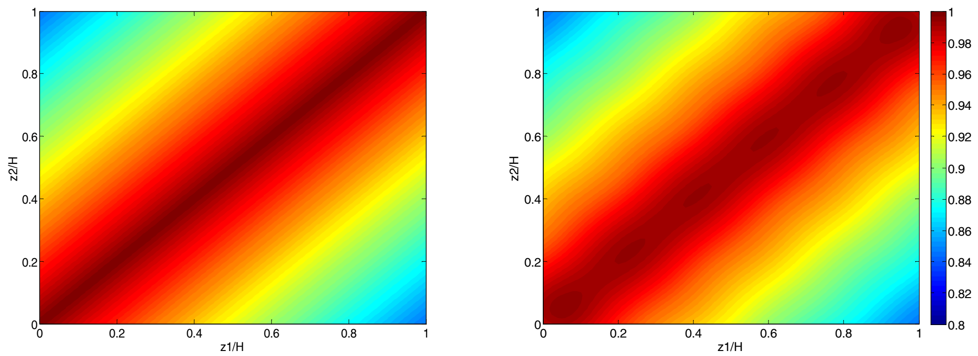


Figure 3: Eigenfunctions $\sqrt{H} f_i(z/H)$, $i = 1, \cdots, 6$.



(a) Exact correlation function (b) Approximate correlation function

Figure 4: Correlation function $C(z_1, z_2)$.

The inhomogeneous half-space is discretized into 20 ordinary layers and 4 CFABC layers (2 CFABC 2-layers) as shown in Figure 2. Each ordinary layer has a depth of 0.1 m. The shear modulus $G_j = G(Z_j) = G\left((z_j + z_{j+1})/2\right)$ for each layer. Two of the CFABC layers are designed for a vertically incident propagating wave with $\theta_P = 0$ and $\theta_S = 0$ and the other two for an evanescent wave with $\alpha_P = 2.146$ and $\alpha_S = 1.073$. A detailed explanation of the construction of the CFABC layers with these parameters is given by Lee and Tassoulas [10].

Using the ordinary and CFABC layers, stochastic responses of the half-space at $(x_r, z_r) = (0, 1m)$ for a unit transverse line load on its surface are calculated. The uncertain displacement is expanded using the polynomial chaos expansion in Equation (7a). The expansion is truncated at the 2nd order term, i.e. $N_d = 2$ in Equation (7a).

The mean and coefficient of variance (COV) of the calculated displacement are compared with Monte Carlo simulations in Figure 5. In the simulation, 10,000 pairs of the standard normal random variables ξ_i's are used. Excellent agreement between the numerical results of this study and the Monte Carlo simulations can be seen.

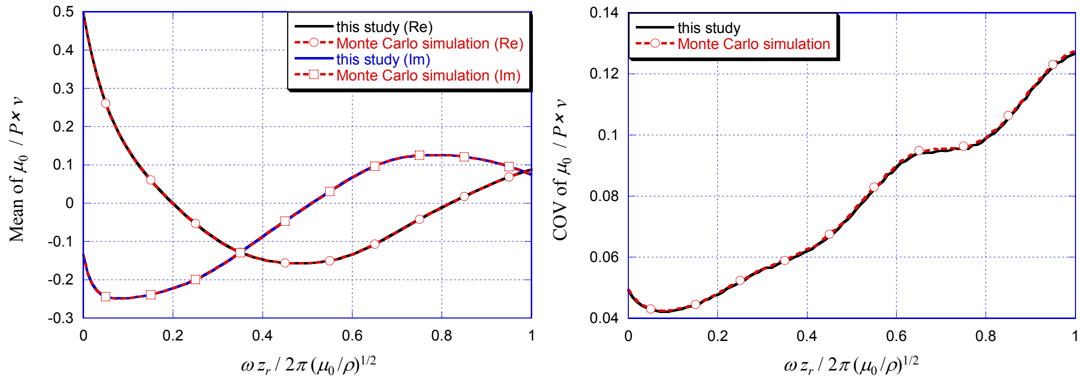


Figure 5: Mean and COV of a displacement.

4 Conclusion

A stochastic thin-layer method is developed for the analysis of wave propagation in an inhomogeneous half-space in antiplane shear. The shear modulus is assumed uncertain and characterised by a random field with vertically varying statistical properties. It is expanded by the Hermite polynomial chaos of a zero-mean and unit-variance Gaussian field with a correlation function. Using the Karhunen-Loève expansion, the Gaussian field can be discretised into independent standard normal random variables. The inhomogeneous half-space is represented by thin layers. The layers include not only ordinary layers but also continued-fraction absorbing boundary conditions for the infinite extent of the half-space. Applying the Galerkin method not only in the spatial domain but also in the stochastic domains, a stochastic thin-layer method for an inhomogeneous half-space in antiplane shear is presented. Using the stochastic methods, dynamic responses of an inhomogeneous half-space subjected to a transverse line load on its surface are obtained and verified by comparison with Monte Carlo simulations. The stochastic methods are found to provide accurate probabilistic treatment of half-space dynamics.

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References

[1] W.T. Thomsom, "Transmission of elastic waves through a stratified solid medium", J. Appl. Phy., 21, 89-93, 1950.

[2] N.A. Haskell, "The dispersion of surface waves on multilayered media", Bulletin of the Seismological Society of America, 43, 17-34, 1953.

[3] E. Kausel and J.M. Roësset, "Stiffness matrices for layered soils", Bulletin of the Seismological Society of America, 71, 1743-1761, 1981.

[4] E. Kausel, "An explicit solution for the Green functions for dynamic loads in layered media", Research Report R81-13, Department of Civil Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts, 1981.

[5] E. Kausel, "The Thin-Layer Method in Seismology and Earthquake Engineering", E. Kausel and G. Manolis, (Editor), Wave Motion in Earthquake Engineering, WIT Press, 193-213, 2000.

[6] M. Shevenels, G. Lomaert, G. Degrande, D. Degrauwe, and B. Schoors, "The Green's functions of a vertically inhomogeneous soil with a random dynamic shear modulus", Probabilistic Engineering Mechanics, 22, 100-111, 2007.

[7] A. Nour, A. Slimani, N. Laouami, and H. Afra, "Finite element model for the probabilistic seismic response of heterogeneous soil profile", Soil Dynamics and Earthquake Engineering, 23, 331-348, 2003.

[8] C.H. Yeh, and M.S. Rahman, M.S., "Stochastic finite element methods for the seismic response of soils", International Journal for Numerical and Analytical Methods in Geomechanics, 22, 819-850, 1998.

[9] M.N. Guddati, "Arbitrarily wide-angle wave equations for complex media", Comput. Methods Appl. Mech. Engrg., 195, 65–93, 2006.

[10] J.H. Lee and J.L. Tassoulas, "Consistent transmitting boundary with continued-fraction absorbing boundary conditions for analysis of soil–structure interaction in a layered half-space", Comput. Methods Appl. Mech. Engrg., 200, 1509-1525, 2011.

[11] S. Sakamoto and R. Ghanem, "Simulation of multi-dimensional non-gaussian non-stationary random fields", Probabilistic Engineering Mechanics, 17, 167-176, 2002.

[12] P.D. Spanos and R. Ghanem, "Stochastic finite element expansion for random media", Journal of Engineering Mechanics, 115, 1035-1053, 1989.

[13] I.S. Gradshteyn and I.M. Ryzhik, "Table of Integrals, Series, and Products", 7th ed., Elsevier, 2007.

[14] J.D. Achenbach, "Wave propagation in elastic solids", Elsevier, Amsterdam, The Netherlands, 1975.

[15] A.C. Eringen and E.S. Suhubi, "Elastodynamics", Vol. II, Academic Press, New York, 1975.