Paper 51

Proceedings of the Eighth International Conference
on Engineering Computational Technology,
B.H.V. Topping, (Editor),
Civil-Comp Press, Stirlingshire, Scotland

Phenotype Building Blocks and Geometric Crossover in Structural Optimisation

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Abstract

Macro or phenotype building blocks (PBBs) contain information at phenotype level. PBBs are either the components of a multi-component system, or different parts of a continuous system with different design qualities and/or evaluation measures. Using PBBs can lead to enhancement of the search efficiency by utilising problem specific search operators and heuristics applied on the PBBs. In order to preserve, propagate and recombine good building blocks efficiently, building blocks should have a low probability of being disrupted by crossover. Therefore, the crossover operation should be designed at phenotype level. Geometric crossover (GCO) is applied on the phenotype rather than the genotype. Identifying PBBs and using GCO, partial fitness can be defined and employed to improve the performance of the search algorithm. Another advantage of using GCO is as a result of its ease of application to non-fixed-length and variable-length chromosomes. Variable-length chromosomes are a common feature of topology optimisation problems when genetic algorithms are employed. GCO can be easily applied to the structural optimisation problems including, topology optimisation without predefining a grid. These two advantages have been demonstrated by implementing GCO in genetic algorithms employed for optimisation of plane trusses supporting distributed loads, two-dimensional steel frames and wind turbine blades.

Keywords: geometric crossover, phenotype building block, partial fitness, topology optimisation, genetic algorithm, structural optimisation.

1 Introduction

The standard explanation of how genetic algorithms (GAs) operate, is often referred as the building block hypothesis [1]. According to this hypothesis, GAs operate by combining small building blocks into larger building blocks. The intuitive idea behind recombination is that combining features (or building blocks) from two good parents will often produce better children.

It is well recognised that the efficiency of a genetic algorithm in exploration and exploitation of the solution space can be improved by incorporating domain-specific knowledge into the algorithm. In many real-world applications, the physics of the problem suggests heuristics that can be incorporated into the search and selection procedures. Whenever GAs are applied to such problems, knowledge about the domain of application should be considered in the design of the reproduction operators as well as the representation and selection. Domain knowledge has been broadly incorporated in selection and reproduction operators. Heuristics or knowledge-augmented operators are tailored for individual applications. The prime consideration in defining these operators is how to implement heuristics in a reproduction operator without jeopardising the exploratory nature of the search.

2 Phenotype building blocks, partial fitness and geometric crossover

In this paper, a phenotype building block refers to a part of solution that contains "interpretable" information; hence it can be assessed by its own merit based on a set of measurable assessment criteria. Assessment criteria of a PBB can be a subset of the assessment criteria of the individual itself, or alternatively, there is at least a known qualitative relationship between the PBB assessment criteria and the individual assessment criteria. A PBB can be a combination of components of a multi-component or discrete system or different parts of a continuous system. A "partial fitness" is assigned to each PBB. Partial fitness can be defined using a non-conflicting subset of the assessment criteria of the PBB.

In the context of engineering design problems: $f_j(\vec{x},\vec{y}) = 0$, ($j = 1,\ldots,q$), in which $\vec{y} = \{y_1, y_2,..., y_q\}$ is the set of design qualities and $\vec{x} = \{x_1, x_1,..., x_p\}$ is the set of design variables, the set of assessment criteria refers to the set of design qualities. Design qualities can be treated as objectives or constraints of the corresponding optimisation problem.

In order to preserve, propagate and recombine good building blocks efficiently, building blocks should have a low probability of being disrupted by crossover. Therefore, either the length of the important building blocks must be short, or alternatively the crossover operator must be tailored for the problem at hand wisely reducing the probability of breaking down good building blocks. In fact, the former is the Holland's original assumption that "the important building blocks are of short length", which is true if segment-based crossover operators such as one- or two-point crossover for bitwise binary strings is used. Using uniform crossover, on the other hand, makes it difficult to propagate good building blocks once they are found.

Using PBBs to incorporate domain-knowledge, the crossover operation should be designed in such a way that disruption of good building blocks is avoided once they are found. PBBs are not of short length in genotype space. Therefore, the traditional crossover operations that operate on the genotype are not suitable for combining the PBBs. Geometric crossover, on the other hand, is applied on the phenotype (real design space) rather than the genotype to avoid disruption of good building blocks.

Generally, a crossover operator, depending on the type of the chromosome encoding and the mechanism of the crossover, may lead to one or a combination of the followings:

- Arithmetic recombination of parents, in which children are in an Euclidean distance of the parents
- Attribute swap between parents, in which the children have common attributes with their parents. A design attribute may be identified by one or more design variables.
- Attribute recombination of parents, in which children may have common attributes with their parents or new attributes resulted by mixing the parents attributes.
- Geometric recombination, in which children have common PBBs with their parents. Depending on the type of the building blocks, a GCO may lead to arithmetic recombination (e.g. new span in the wind turbine blade example of Section 4), attribute swap (e.g. constructing a child by the axes of one parent and bays of another in the plane frame example of Section 3), and attribute recombination (e.g. weight of child in the truss example of Section 5).

The application of GCO is not limited to the structural optimisation. “Cut and splice” is a type of crossover which is applied on the real design space. This type of crossover has been employed for pattern recognition and structural optimisation of atomic clusters problems. Cross et al [2] and later Myers and Hancock [3] applied a similar concept for pattern recognition problems. They combined the solutions by physically dividing the graphs into two disjoint subgraphs. Their results show that their crossover operation improves the search convergence speed. More recently, Nazim and Erkoc [4] and Froltsov and Reuter [5] applied similar concept in structural optimisation of atomic clusters. In their genetic algorithm a new structure is generated by cutting existing cluster geometries into two halves and then recombining the halves of different configurations. It should be noted that in none of these works the concept of partial fitness has been used in the process of parent selection.

3 Application to planner frames

Figure 1, shows a typical $m \times n$ two-dimensional structural frame made of $n_c = (m+1)n$ standard columns and $n_b = mn$ standard beams. Each column and beam can be identified by an index, referring to its standard code. Using an indexed coding, the chromosome of this frame can be defined by a string of length $n_c + n_b$. For this chromosome representation, beams and columns are genotypes. However, one can define phenotypes building blocks as axis (four columns), bay (four beams) and storey (four columns and three beams). Each of these three PBBs can be assessed based on a subset of the frame assessment criteria (e.g. weight, lateral storey drift, beam deflections, beam and column maximum stresses, column critical bucking load, etc). Each frame can be made of storeys or combination of axes and bays.

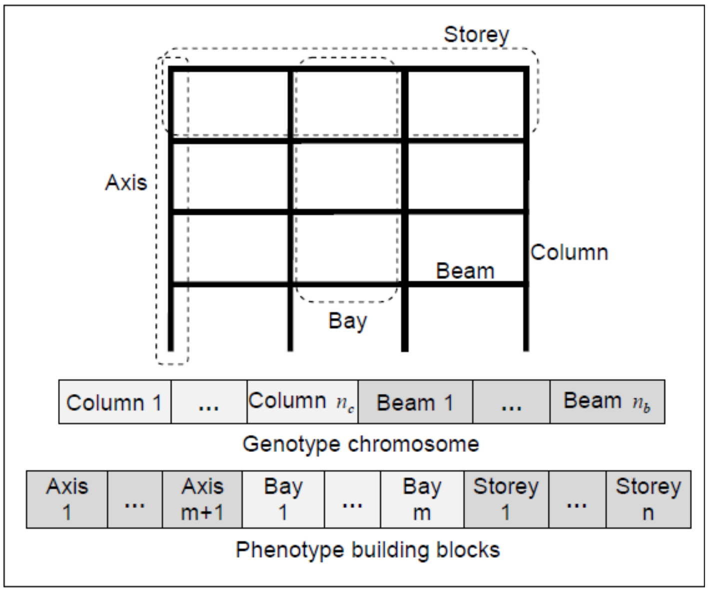

Figure 1: A typical $m \times n$ plane frame and its PBBs.

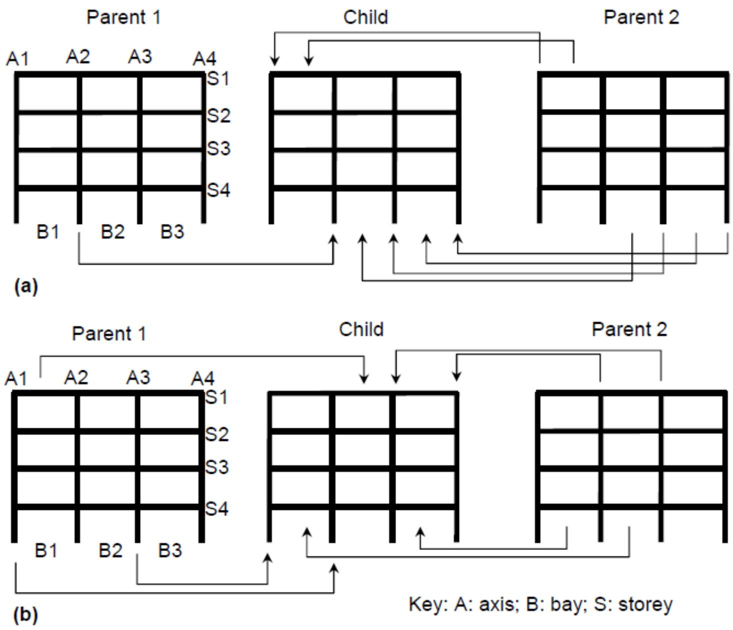

Figure 2: Geometry crossover of the two-dimensional frame problem using PBBs: (a) homologous and (b) non-homologous.

Figure 2 shows the PBB exchange in a GCO of the frame of Figure 1. Since there is more than one type of each PBBs, both homologous and non-homologous crossover can be applied to two parent frames.

In order to investigate the effect of implementing PBB, partial fitness and GCO in the design of frames, one of the design case studies of reference [6] is adopted in which the optimum allowable stress design of steel frames subjected to various loadings and load combinations under constraints of AISC–ASD specification is desired. Design variables are the element section sizes from the available W shapes of a standard list. For a steel frame consisting of N_m members that are collected in N_d design groups (number of design variables), the objective is to find a vector of integer values I (Eq. (1)), representing the sequence numbers of steel sections assigned to N_d member groups:

$$I^T = [I_1, I_2, ..., I_{N_d}] \tag{1}$$

for minimizing the weight of the frame, expressed as:

$$W = \sum_{i=1}^{N_d} \gamma_i A_i \sum_{j=1}^{N_t} L_j \, , \tag{2}$$

subject to a series of constraints, as formulated and detailed in [6], on combination of axial and flexural stress, column Euler stress, column slenderness ratio, shear stress, ductility, beam deflection serviceability, storey drift serviceability and constructability.

In Equation 2, A_i and γ_i, respectively, represent the area and the unit weight of the steel section adopted for the member group i, N_t stands for the total number of members in group i and L_j is the length of the member j which belongs to group i.

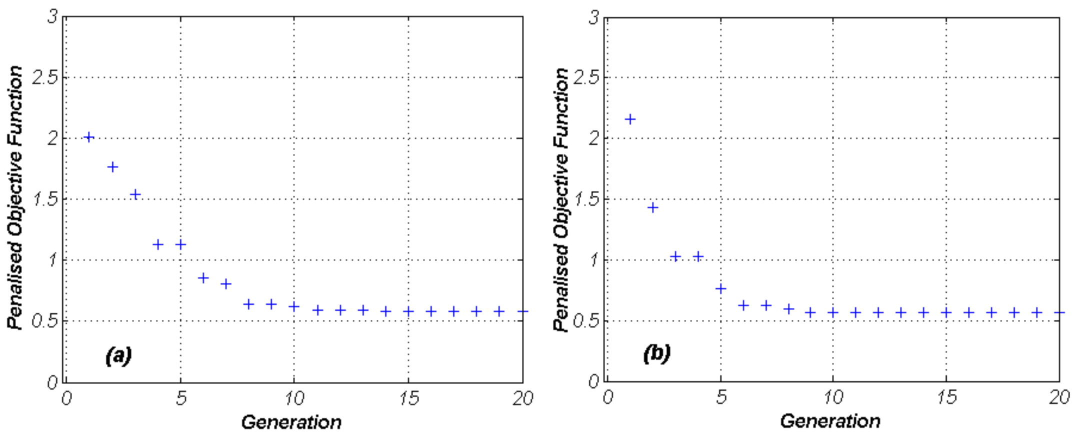

Figure 3: Search history using (a) bitwise crossover converged at generation number 12 to 0.583 and (b) geometric crossover converged at generation number 9 to 0.568

In order to investigate the performance of employing GCO, PBB and partial fitness in optimisation of frames, starting with the same initial population, two types of crossover is used: (i) GCO with partial fitness $PF = 1/weight$ for all types of PBBs, and (ii) conventional bitwise crossover. Figure 3 shows the search histories for the two runs. Evidently, when using GCO, the search converges faster (generation number 9 versus 14) to a better solution (penalised objective function of 0.568 versus 0.583).

4 Application to wind turbine blades

Aerodynamic design of wind turbine blades includes optimisation of the topology of the blade. Parameters such as rotor radius and the span-wise distributions of the chord length, pretwist and aerofoils define the topology of the blade and are treated as design variables. A wide range of parameters construct the set of design qualities (objectives/constraints). Normally, the average annual power yield or the power coefficient at the design wind speed are treated as optimisation objectives, while constraints are applied on the aerodynamic loads, blade internal forces, blade weight, fatigue life, etc .

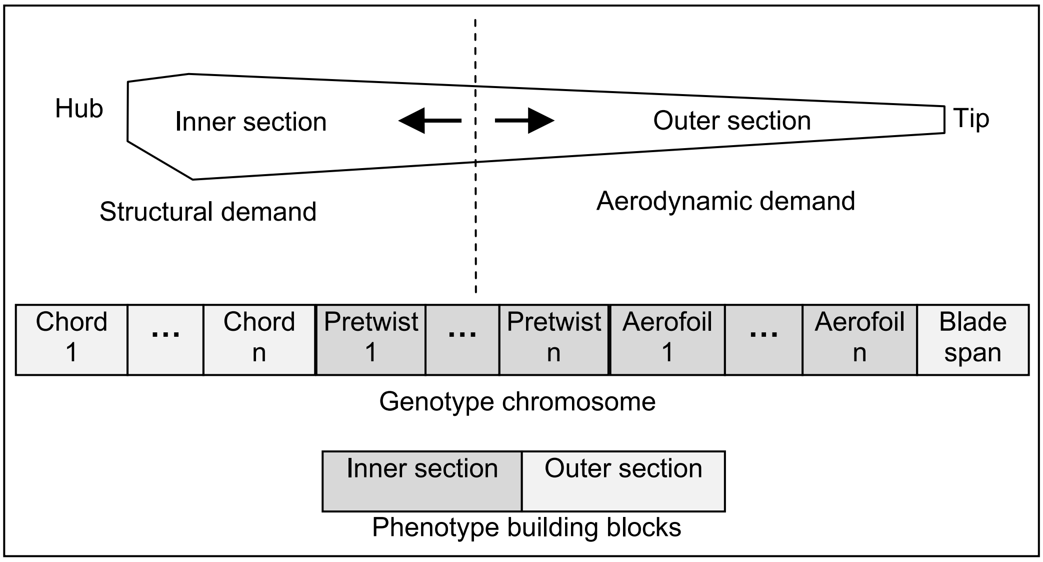

Figure 4: Wind turbine blade and its PBBs

Figure 4 shows a wind turbine blade. Using n design (precision) points for the distributed design variables (chord length, pretwist and the aerofoil code), the total number of design variables will be $3n+1$. Employing a mixed real-number/indexed encoding, each individual can be defined by a chromosome of length $3n+1$. The assessment criteria for the blade can be defined as the weight of the blade, maximum flap stress and the average power production corresponding to a given wind probability density function. Blade is a continuous structure, but its inner and outer sections have different functions and can be defined as two PBBs. The outer section of the blade is aerodynamically more efficient. That is, the power is mostly produced by the outer section of the blade. Whilst the inner section experiences greatest internal forces and the design is driven based on the structural demand. Since the inner and outer sections have different functions they can be assessed separately. The inner section can be assessed based on, for example, the maximum flap stress or the weight of the blade, while the outer section can be assessed based on the produced average power. In contrast to the frame example, there is no sharp boundary between the wind turbine blade PBBs.

The genotype chromosome is a string of real numbers as well as indexes, as shown in Figure 5.a. The design variables can be either distributed (chord, pretwist, aerofoil thickness and aerofoil code) or single value (blade span). The design variables can also be categorised as either continuous (blade span, chord, pretwist and aerofoil thickness) or discrete (aerofoil code). Figure 5.b demonstrates the GCO, assuming that parents P_1 and P_2, respectively, have better performance in terms of design qualities corresponding to the inner and outer segments of the blade. The cut point is a randomly selected real number between 0 and the span of the blade. Since there is only one of each type of PBBs, only homologous crossover is possible.

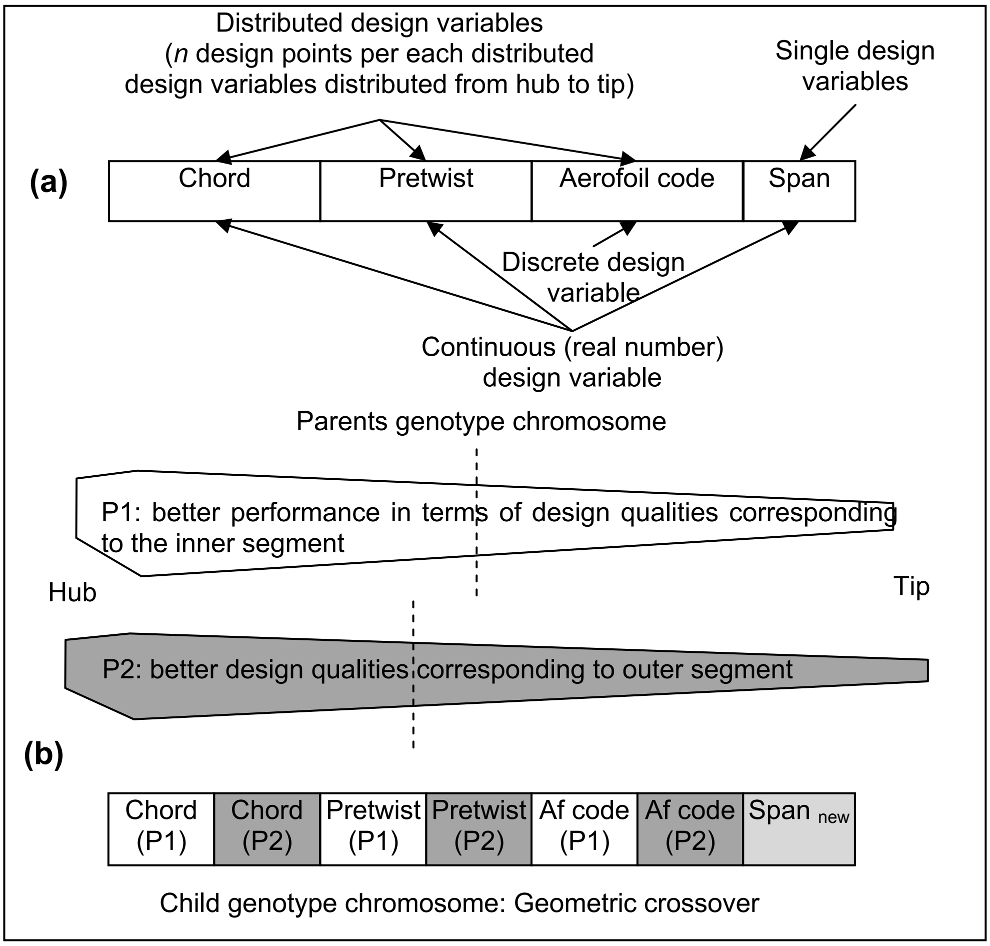

Figure 5: Wind turbine blade (a) Genotype chromosome, (b) GCO

Having the radial coordinate normalised by span length ($r^* = r / span$), the cut point is a randomly selected precision point r_c^*. The cut divides each parent blade into two parts. The distributed design variables of the child blades are formed by those of the left and right hand sides of each parent blade. A repair operation is also required to retain the continuity of the distributed design variable.

Figure 6 illustrates the process of forming a distributed design variable (here the pretwist distribution β_0) of a child from a pair of parents. The repaired pretwist is obtained by multiplying the unrepaired pretwist by the left and right multipliers $M_L(r^*)$ and $M_R(r^*)$.

$$
\left[\beta_0(r^*)\right]_{C_1,R} = \begin{cases} \left[\beta_0(r^*)\right]_{P_1} M_L(r^*) & if \quad 0 \le r^* \le r_c^* \\ \left[\beta_0(r^*)\right]_{P_2} M_R(r^*) & if \quad r_c^* < r^* \le 1 \end{cases} \tag{3}
$$

where, subscripts C_1, P_1, P_2 and R stand for child 1, parent 1, parent 2 and repaired, respectively. $M_L(r^*)$ and $M_R(r^*)$ are the left and right segments of a multiplier curve. The multiplier curve for child 1 is a linear curve between 1 at $r^* = 0$ and $\frac{\lfloor \beta_{0,c} \rfloor_{C_1,R}}{\lfloor \beta_{0,c} \rfloor_{P_1}}$ at the cut point; and $\frac{\lfloor \beta_{0,c} \rfloor_{C_1,R}}{\lfloor \beta_{0,c} \rfloor_{P_2}}$ at the cut point r_c^* and 1 at $r^* = 1$ as shown in Figure 6.c. The pretwist at the cut point r_c^* is denoted by $\beta_{0,c}$. The repaired pretwist at the cut point is a combination of the left and right values proportional to the length of the left and right segments respectively. That is, the repair process has less effect on the segment with longer length.

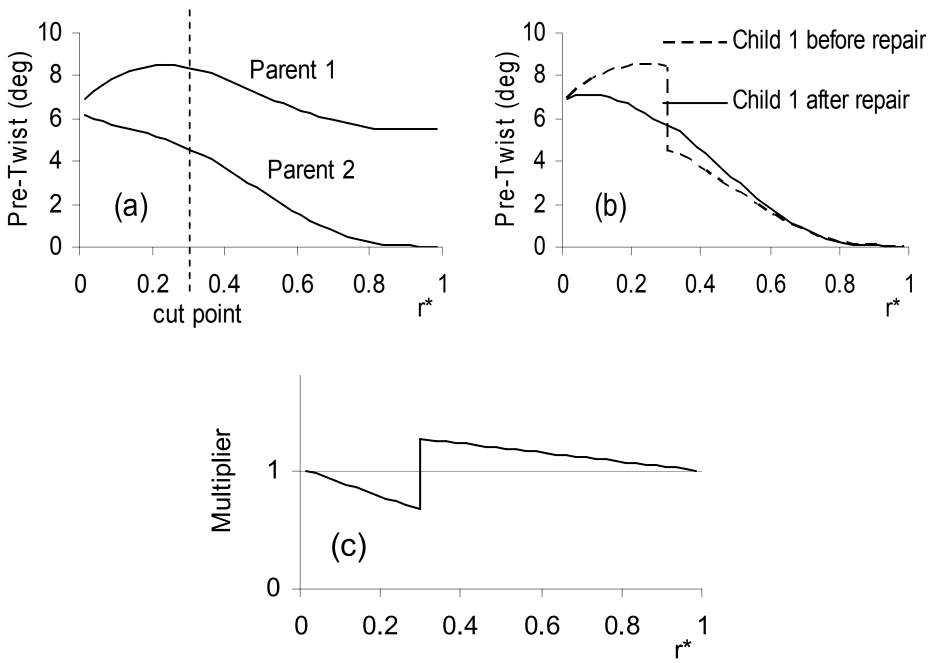

Figure 6: Pretwist formation of a child blade; child 1 is formed based on the left segment of parent 1and the right segment of parent 2.

Single value design variables, for example the blade span of child 1, R, is the combination of those of parent blades in a weighting sense.

$$R_{child\,1} = r_c^* R_{parent\,1} + (1 - r_c^*) R_{parent\,2} \tag{4}$$

Interchanging indices 1 and 2 in the above equations, the second child will be formed.

In order to investigate the performance of employing GCO, PBB and partial fitness in optimisation of wind turbine blades, blade of AWT27, a 2-bladed stall regulated wind turbine, is selected to be optimised for the pretwist and chord distributions. An initial population of size 100 is generated randomly. Blades of the initial population have the same span and aerofoil distribution as the baseline. Using the same initial population, two types of crossover is used: (i) GCO with partial fitnesses $PF_1 = P_{av}$ for the outer section and $PF_2 = 1/weight$ for the inner section; and (ii) arithmetic crossover. For GCO the pretwist of the produced child is obtained by

$$\left[\varphi(r^*)\right]_{C_1,R} = \begin{cases} \left[\varphi(r^*)\right]_{P_1} M_L(r^*) & if \quad 0 \le r^* \le r_c^* \\ \left[\varphi(r^*)\right]_{P_2} M_R(r^*) & if \quad r_c^* < r^* \le 1 \end{cases} \tag{5}$$

in which, φ represents pretwist β_0 and chord c.

In the case of arithmetic crossover the pretwist and chord distributions of the child is given by:

$$\varphi_{child} = \lambda \varphi_{parent\,_1} + (1 - \lambda) \varphi_{parent\,_2} \tag{6}$$

in which, again, φ represents pretwist β_0 and chord c and λ is a random number.

In performing arithmetic crossover, a roulette wheel constructed based on the fitness $fitness = P_{av}$ is employed, while in the case of GCO, two roulette wheels are constructed based on the partial fitnesses defined above. These roulette wheels are employed to select the parents.

Starting with the same initial population, 2000 crossover operations of each type is carried out. Results presented in Figure 7 show an improvement of 11.6% in the average power of the population when GCO is used versus an improvement of 8.6% when arithmetic crossover is used. Results also show that the population quality in terms of the weight improves slightly more when using GCO (12.2% versus 11.7%).

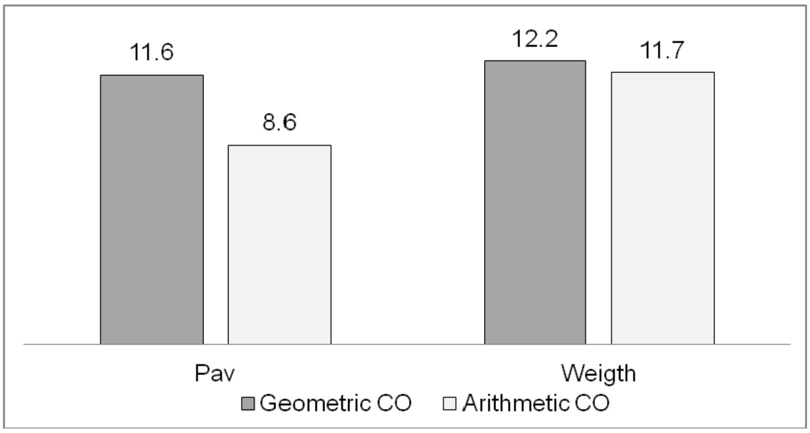

Figure 7: Percent improvement in the qualities of the population: geometric versus arithmetic crossover

5 Application to trusses supporting distributed loads

Figure 8 shows a two dimensional truss, constructed of a string of triangular panels and subjected to a distributed load $q(x)$. The topology of this truss, including the number of panels, is to be optimised with the objective of minimising the weight of the truss, subject to a constraint on the load conversion expediency (LCE), a parameter defined based on the structural properties of the load domain [7]. Assuming that members are made of the same material and that their cross sectional areas are proportional to their internal forces, minimisation of the weight of the truss is equivalent to minimising the objective function h, defined as

$$h = \sum_{i=1}^{m} l_i\left(\vec{X}\right) P_i\left(\vec{X}\right) \; ; \; i \in \{1,2,...,m\} \tag{7}$$

subject to the constraint

$$LCE \geq LCE_c \tag{8}$$

where, LCE_c is the smallest permissible LCE , l_i and P_i ($P_i \neq 0$) represent the length and the internal force of member i respectively and m is the number of members in the truss. In the case of zero members ($P_i = 0$), P_i will be replaced with the smallest internal force in the truss members. In the above equation $\vec{X} = \{n, x_j, y_j\}$, ($j \in \{1,2,...,n\}$), stands for the vector of design variables, parameter n is the total number of nodes, and x_j and y_j are the coordinates of node j respectively. More details on the formulation of the problem can be found in reference [7]. Using a mixed integer-real number encoding, the genotype chromosome of this truss will have a variable length of $2n-3$, where n is the total number of nodes.

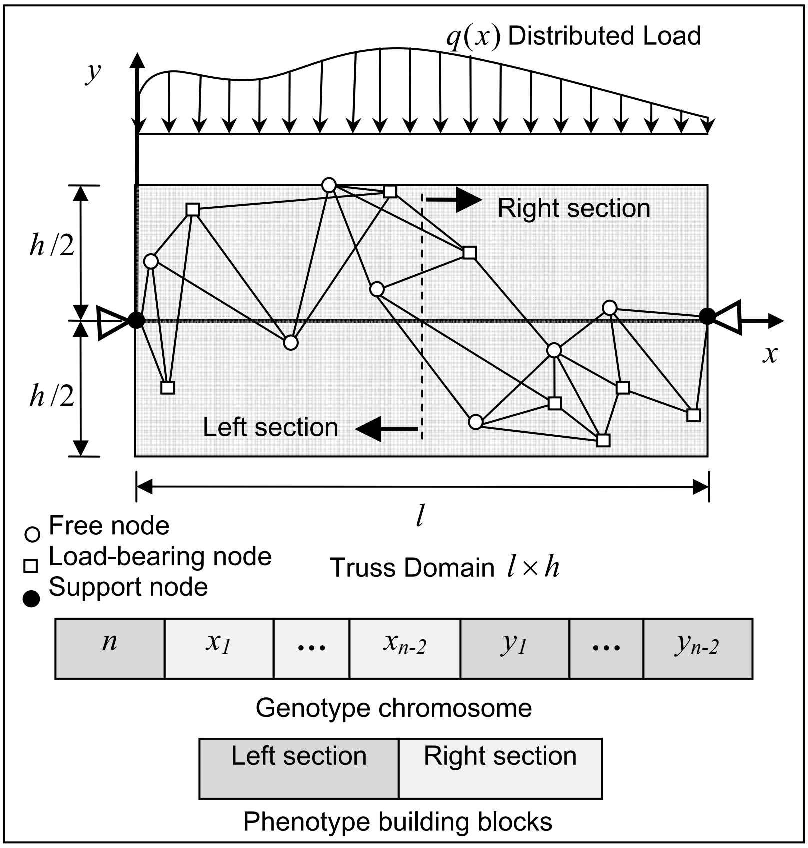

Figure 8: Plane truss and its PBBs

In the first two examples, the PBBs were either, configurationally, easy to identify as in the case of the frame or had different functions as in the case of the wind turbine blade. However, none of these conditions are necessarily required to be valid for PBBs.

This truss can be viewed as a discrete system similar to the frame example. However, since it is to be optimised for the topology, it does not have a fixed topology through optimisation process. As the PBBs are fixed, the biggest possible PBB is a triangular panel. Obviously, the smaller the PBB, the closer it becomes to the genotype form. This is not aligned with the original philosophy behind using PBBs for incorporating knowledge into the search process. An alternative approach is to treat the truss as a continuous system similar to the wind turbine blade. In this way each truss can be defined based on two PBBs: its left and right sections. In the wind turbine blade problem, the phonotype building blocks are identified based on their different functions and have different assessment criteria. However, in the truss problem, the functions of the left and right sections of the truss are the same; hence, their assessment criteria will be the same.

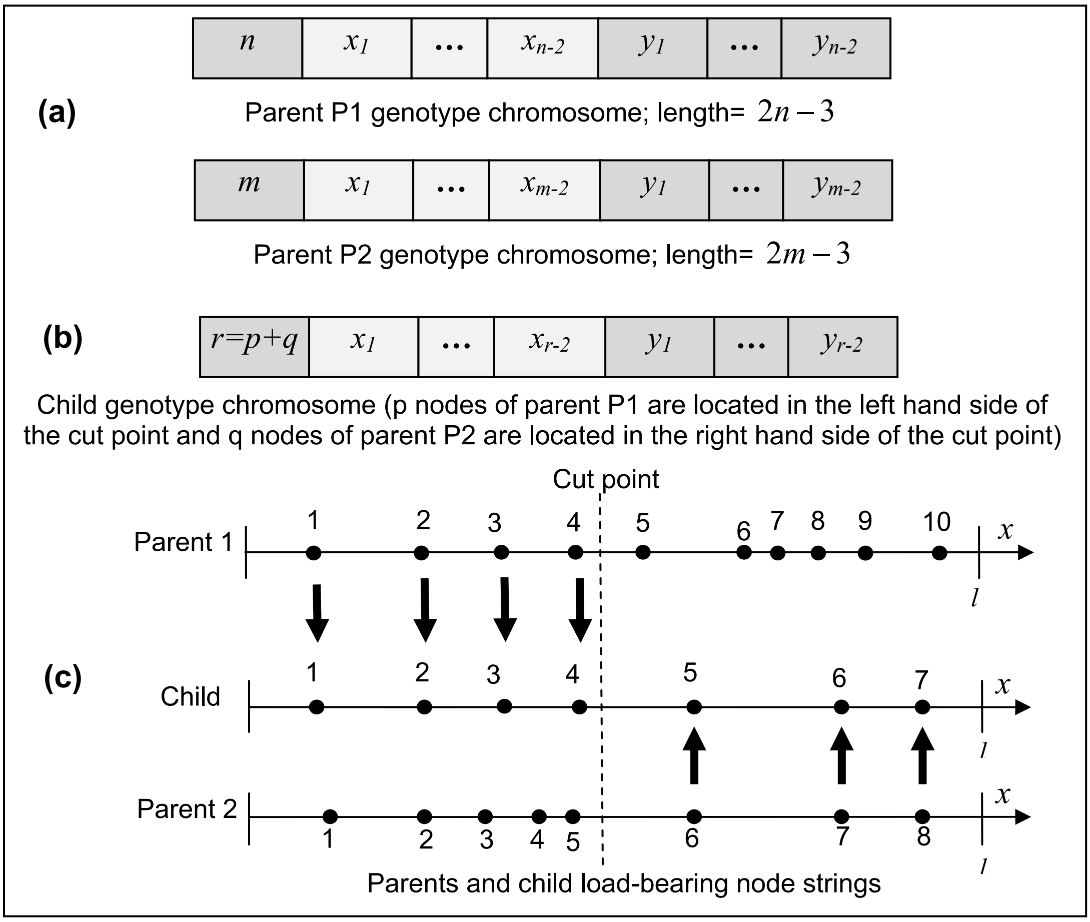

Figure 9: Plane truss (a) parents genotype chromosome, (b) child genotype chromosome after GCO, (c) illustrative example showing the change in the number of panels due to GCO (only the x-coordinate of the load-bearing nodes is shown)

The GCO of the truss problem is illustrated in Figure 9. Figures 9.a and 9.b show the genotype chromosome of two parents and the offspring made by a GCO operation. A geometric cut divides each parent truss into two parts. Assuming that the left section of parent P_1 is better than the left section of parent P_2 and the right section of parent 2 is better than the right section of parent 1, the left section of the first parent is combined with the right section of the second parent to form a child truss. A repair operation also might be required at the cut point to make the child truss kinematically stable. The location of the cut point and the topology of the parents influence the topology of the offspring. Generally, the length of these three chromosomes can be different. That is, the offspring can have a different topology from its parents. The length of the offspring chromosome is $r = p + q$, where p and q are the number of nodes on the left section of the first parent and the right section of the second parent, respectively. Figure 9.c shows the x-coordinate of the load-bearing nodes of two parents and their offspring. This figure illustrates how two chromosomes with different lengths are combined and make a new chromosome with a different length.

Figure 10 shows the search histories of the above optimisation problem once using GCO and once applying conventional arithmetic crossover. Evidently, employing GCO and partial fitness improves the search convergence rate. The fitness is defined as the product of the reciprocal of the weight of the truss and a penalty function:

$$fitness = \frac{1}{h} p \tag{9}$$

The penalty function p is defined as:

$$p = \min\left\{1, \frac{LCE - LCE_0}{LCE_c - LCE_0}\right\} \tag{10}$$

where, LCE_0 is the minimum possible value for LCE.

The partial fitnesses used for the left and right hand sides of the truss are defined as the reciprocal of the weight of the left and right hand sides of the truss respectively.

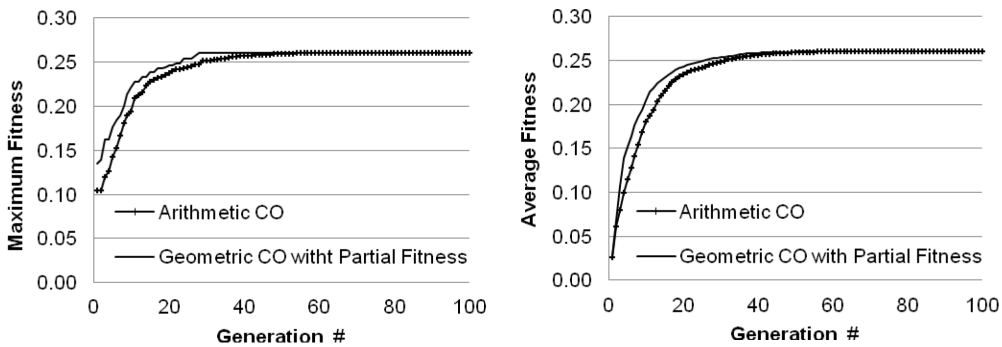

Figure 10: Search histories in the qualities of the population: geometric versus arithmetic crossover

6 Conclusion

By identifying PBBs and using GCO, partial fitness can be defined and employed to improve the performance of the search algorithm. It is shown that these concepts can be applied to a wide range of structural optimisation problems with different characteristics. These concepts can be applied to both, discrete and continuous structures. PBBs may or may not have sharp boundaries. PBB can be defined as a combination of genotype building blocks of a discrete system or it can be identified based on different functions of different segments of a continuous system. Partial fitness can be defined based on either the individual assessment criteria or some new assessment criteria. There is no restriction on the type of the design variables involved in the problem (indexed, real value, integer, distributed, single value, continuous and discrete). Another advantage of using GCO is due to its ease of application to non-fixed-length and variable-length chromosomes. Variable-length chromosomes are a common feature of topology optimisation problems when using genetic algorithms. Hence, GCO can be easily applied to structural optimisation problems including topology optimisation without predefining a grid.

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