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Modal-Space Based Solutions including Geometric Nonlinearities for Flexible Multi-Body Systems

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Abstract

Multi-body system (MBS) software packages have been originally developed for simulation of rigid body system dynamics. However, many physical systems require consideration of the behaviour of flexible bodies in the MBS in order to meet the requirement for improved simulation accuracy. In order to retain high numerical efficiency, model reduction techniques are addressed, particularly modal space transformations. The approach is limited to linear deformations with respect to the body-fixed reference frame. However, parts of certain physical systems may exhibit deformations characterized by non-negligible geometrical nonlinearities. This paper discusses techniques to account for geometrical nonlinearities in modal space based solutions when moderately large deformations are handled.

Keywords: multi-body systems, modal-space solution, geometric nonlinearity.

1 Introduction

Over the past few decades, a number of Multibody-Simulation (MBS) software packages, such as ADAMS and SIMPACK, have been developed with the aim to assist engineers to model, simulate, analyze and design various types of complex mechanical systems. The original development aimed at analysis of nonlinear dynamical behaviour of rigid-bodies. Today, however, the request for the features of MBS-software is much more demanding. Not only are they supposed to enable interdisciplinary modelling and analysis, but consideration of elastic behaviour becomes in many cases a high priority demand [1]. The advantages this consideration offers include possibility of analyzing interactions between deformations of elastic bodies and the behaviour of the rest of the system, obtaining stress states of the system parts of interest, which further allows assessment of the suitability of the part design for the foreseen life-time, etc.

The basic idea of incorporating elastic properties of bodies in MBS software consists of decomposition of overall motion into large rigid-body motion and small deformations that are appropriately described in a local (body-fixed) reference frame. Based on this idea, the elastic properties of flexible bodies are described by the linear stiffness matrix given with respect to the local reference frame of the body. Further improvement of numerical efficiency is achieved by transformation from nodal DOFs to modal DOFs and, hence, the solution is obtained in modal space. In many cases, however, certain parts of considered mechanical systems undergo deformations that are described as moderately large. Such deformations are characterized by nonlinear effects and are not adequately described by linear approach. Geometrical nonlinearities can have their cause in stress stiffening effects, relatively large configuration changes with respect to the floating reference frame and, finally, contact. In general, it is a combination of these causes that results in geometrically nonlinear elastic behaviour. The existing modal space based solutions that account for stress stiffening effects are shortly discussed in the paper. Particular attention is, however, given to the proposed approach that aims at improved accuracy of predicting deformed configuration when deformational behaviour involves moderately large local rotations. At this point, the approach is tested for beam-like structures.

2 Flexible bodies in MBS dynamics

A considerable amount of work has been dedicated to development of formalisms to simulate flexible bodies in MBS dynamics. The common approach is to describe the deformation of flexible body with respect to a body-fixed (floating) reference frame. The approach allows distinction between large rigid-body motion and small deformational motion. Hence, nonlinearities originating from the large rigid-body motion can be considered separately, which was the primary objective of software packages for MBS dynamics. But even with this approach, keeping FEM models in their full extent (nodal description) would be numerically very expensive and time-consuming in most cases of MBS dynamics.

A model reduction is performed with the aim of reducing the computational burden. The widely accepted approach within the MBS dynamics relies on modal superposition technique, according to which the nodal displacements $\boldsymbol{u}$ are given as:

$$\boldsymbol{u}(t) = \sum_i q_i(t)\boldsymbol{\varphi}_i \,, \tag{1}$$

where q_i are the modal coordinates and $\boldsymbol{\varphi}_i$ the mode shapes. Hence, it implies that orthogonal mode shapes, computed prior to simulation, become the degrees of freedom in terms of which the elastic behaviour of the body is described. Not only is the number of degrees of freedom in this manner significantly reduced, but the equations describing elastic behaviour are also decoupled, i.e. the generalized mass ($\boldsymbol{M_m}$), damping ($\boldsymbol{C_m}$) and stiffness ($\boldsymbol{K_m}$) matrices are diagonal [2]:

$$\boldsymbol{M_m}\ddot{\boldsymbol{q}} + \boldsymbol{C_m}\dot{\boldsymbol{q}} + \boldsymbol{K_m}\boldsymbol{q} = \boldsymbol{F_m}(t)\,, \tag{2}$$

where $\boldsymbol{q}$ is the vector of modal coordinates and $\boldsymbol{F_m}$ denotes the external forces in modal space. The quality of the obtained solution strongly depends on the mode shapes used in the simulation. The commercial software package ADAMS uses the Component Mode Synthesis (CMS) technique, particularly the Craig-Bampton method. The method requires to partition the flexible body degrees of freedom (DOFs) into boundary DOFs and interior DOFs, the former belonging to the nodes of the FE-model that the user wants to retain in the simulation model mainly for the purpose of defining (kinematic or dynamic) boundary conditions. In the next step, the method requires to determine two sets of modes: 1) *constraint-modes*, which are static shapes obtained by giving each boundary DOF a unit displacement, while all other boundary DOFs are fixed; 2) *fixed-boundary normal modes*, which are obtained by fixing all boundary DOFs and computing an eingensolution. Since the so-obtained Craig-Bampton modes are not an orthogonal set of modes, they are not suitable for direct use in MBS dynamics and are, therefore, orthonormalized prior to simulation. Compared to classical approach based solely on eigenmodes with predefined boundary conditions (used with the FEM), the Craig-Bampton modes offer greater flexibility of the model with respect to applicable boundary conditions.

The modal approach is obviously suitable for small deformations with respect to the body-fixed reference frame. The deformations have to be small enough so that the linear formulation yields a satisfactory accuracy. But, as already emphasized, in certain cases, the limits of “linear” deformations are exceeded and the accuracy obtained by the linear approach is deteriorated beyond an acceptable level. At that point, there are two requirements imposed onto flexible-body formulation that are not easy to conciliate. The first one is to retain the high numerical efficiency offered by the modal superposition technique. The second one is to improve the accuracy of the obtained solution so that moderately large deformations are accounted for up to certain degree. A solution can be obtained by certain modifications of the modal based approach. In the next section, already available solutions based on the geometric stiffness matrix are addressed in short. Thereafter, the attention is focused to a newly proposed solution that aims at more accurate description of deformed configuration upon deformations involving moderately large local rotations.

3 Geometric stiffness based solutions in modal space

A geometric stiffness based solution is available in the commercial software package SIMPACK [3] for beam structures. The applied approach differentiates between forces, with respect to which the structure is quite flexible (i.e. forces that may cause large deformations) and forces, with respect to which the structure is rather stiff [4]. The latter forces may be quite large, thus inducing significant stresses in the structure, but not large deformations (e.g. tensile force acting upon a beam). The large stresses influence the stiffness characteristics of the structure and the influence is accounted for through the geometric stiffness matrix. Since the deformations are small, the stresses are assumed to be linearly dependent on acting forces. Hence, for each single load case acting on the body, the corresponding stress state due to unit excitation (force or moment) is computed and, furthermore, the geometric stiffness matrix that corresponds to the stress state caused by the unit excitation is computed.

The so-computed geometric stiffness matrix is scaled with the actual value of the corresponding load. The overall geometric stiffness matrix is then approximated as a superposition of geometric stiffness matrices due to all acting loads.

The authors of the paper have developed a modification of the approach [5], in which a unit deformation according to each of the retained modes is imposed as an excitation to get single geometric stiffness matrices that are, furthermore, scaled by the actual modal coordinates and superposed to approximate the overall geometric stiffness matrix.

The approach is limited to specific cases of deformation characterized by significant stress stiffening effects, but small configuration changes.

4 Method based on rotation of modal displacements

Geometrically nonlinear behaviour may be caused by large configuration changes over the course of deformation, particularly by relatively large rigid-body rotation of structural sub-domains. With certain structures, it is possible to make distinction of sub-domains that can perform relatively large rigid-body rotations with respect to each other. Depending on deformation itself, some of those cases can be handled well with MBS programs that implement floating reference frame approach in combination with modal superposition technique for flexible bodies. Namely, the floating reference frame accounts for a single averaged rigid-body rotation and this might suffice to improve the accuracy of the predicted deformational behaviour, but it does not hold for a general case.

The authors propose a method that aims at improving the prediction of the deformed structure configuration for the above described type of deformation. The following description of the method proceeds without the floating reference frame and the same is valid for the below given examples. This represents a conservative choice to test the method. The method is just as well applicable in a formulation that incorporates the floating reference frame and this would probably allow the coverage of a wider range of deformations.

The idea behind the method is rather simple and consists in two steps. Within the first step, the displacements of the structure, as a consequence of its flexibility, are determined by means of modal superposition technique. This motion, however, is still partly a rigid-body motion. The amount of rigid-body motion is different for different particles of the structure. In the second step, a sort of averaged rigid-body rotation is determined and it is used to rotate the modal space based displacements of the body. The improvement of accuracy, with which the deformed configuration is predicted, depends on the complexity of deformation and possibility of adequate averaging of the rigid-body rotation contained in modal based displacements.

The above given description of the method can be conveniently illustrated on a rather simple structure. It is a clamped beam structure, given in Figure 1. The eigenmodes of the structure are used as degrees of freedom. They are obtained based on linear analysis. Observing a deformation according to the first eigenmode, it would appear similar to the deformation depicted in Figure 1a. The predicted displacement of the beam tip would be such that the tip remains on the line perpendicular to the original configuration, whereas in reality, the tip of the beam would

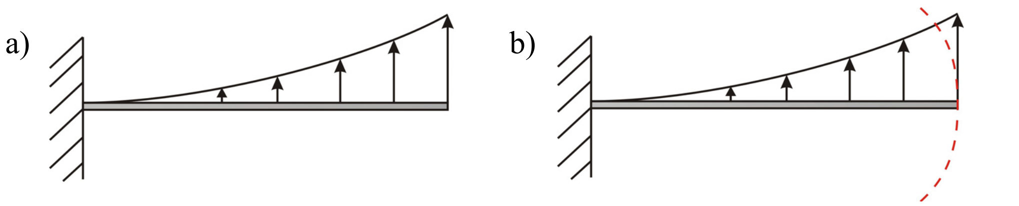

Figure 1: Clamped beam: a) deformation according to the first mode;
b) actual beam tip motion

perform a motion similar to the dotted line in Figure 1b. The method aims at a better approximation of the actual deformed configuration and its essence is depicted in Figure 2. Hence, the displacements are firstly calculated as originally yielded by the modal space based solution. In this case that would be the displacement according to the first eigenmode. Then, an averaged rotation performed by the structure during the motion is determined. It is symbolically represented in Figure 2a by angle α. This amount of rotation is further used to rotate the displacements computed in the first step, as depicted in Figure 2b.

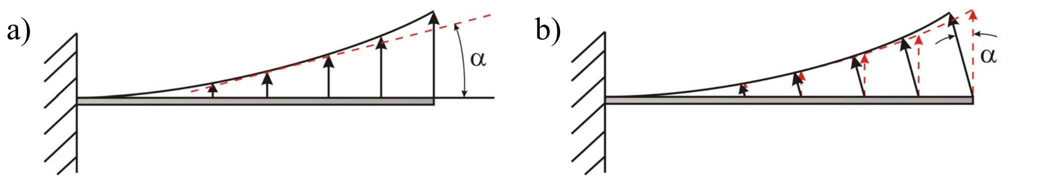

Figure 2: Clamped beam: a) averaged rotation of the beam;
b) rotation of displacements

Another important aspect of the approach needs to be given attention. As the structure deforms, the external forces might change their lines of action, whereby the same direction is kept (Figure 3). The linear analysis does not recognize this effect, since the reference configuration is always the initial one. Geometrically nonlinear analysis, however, takes the change into account. For an analysis based on modal superposition technique, the reference configuration is the initial one. However, once the deformed configuration is obtained using the above described approach, it is easy to determine the new lines of action of external forces (Figure 3a). This further allows the computation of corrective moments that compensate for the change of the line of action caused by displacements $\boldsymbol{u_F}$ of the point at which the force

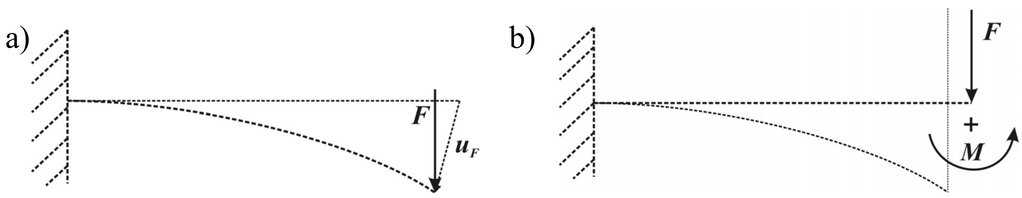

Figure 3: Clamped beam: a) force acting upon the deformed configuration;
b) force acting the upon initial configuration and corrective moment

$\boldsymbol{F}$ is applied when the computation is performed based on the initial configuration (Figures 3a and 3b). The corrective moment $\boldsymbol{M}$ is simply given as:

$$\boldsymbol{M} = \boldsymbol{u_F} \times \boldsymbol{F} , \tag{3}$$

and applied as an additional external excitation.

5 Examples

The described approach is a heuristic one. It is expected to yield an improvement in accuracy over the pure modal space solution for beam-like structures, particularly those that experience motion similar to the one characteristic for the above given clamped beam. Therefore, the method will be first tested for the very same structure that was used for illustrative description of the approach. However, the approach is designed having in mind more complex structures, the geometry of which distinguishes several sub-domains that can perform relatively large rigid-body rotations with respect to each other. The idea is to apply the approach to single sub-domains. This will be demonstrated in the second example that involves a rear car axle. The accuracy of predicting deformed configuration caused by static loading will be used as a criterion to test the approach. The reference solutions will be linear and geometrically nonlinear solutions from ABAQUS, but solutions from the MBS software package ADAMS will also be given for the rear car axle.

5.1 Clamped beam

The example has already been elaborated above. It is the clamped beam with specific dimensions given in Figure 4 and made of steel (E = 2×10^{11} N/m^2, ν=0.3). The acting force has the magnitude of F = 10 kN and it is chosen so that non-negligible geometrically nonlinear effects are obvious in deformational behaviour.

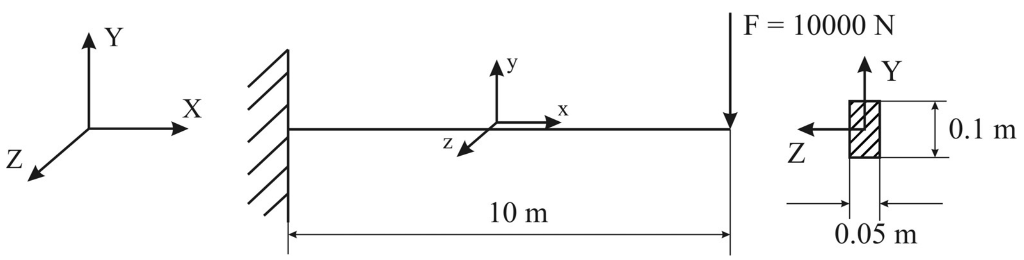

Figure 4: Clamped beam with dimensions and excitation force

For the computation in modal space, the first 10 eigenmodes have been used, although it is obviously the first eigenmode that plays the major role in the considered case. A local coordinate system has been defined to determine the average rigid-body rotation. It is defined for the deformed configuration so that its xy-plane is determined by three points: the clamped end of the beam, the free end of the beam in the original/undeformed configuration and the free end of the beam in the current/deformed configuration. The x-axis is taken to lie along the line defined by the clamped end of the beam and the free end of the beam in the current configuration. The z-axis is easily determined by cross-product of the x- and y-axis. Performing the computation with the so-chosen local reference frame, the results shown in Figure 5 (beam tip displacement in the global X-direction) and Figure 6 (beam tip displacement in the global Y-direction) are obtained. A very high agreement of the geometrically nonlinear results from ABAQUS and results from the proposed method can be observed. It should be noticed that the linear result yields zero displacement of the beam tip in the global X-direction. Hence, this result is given in Figure 5 as congruent with the abscissa of the diagram.

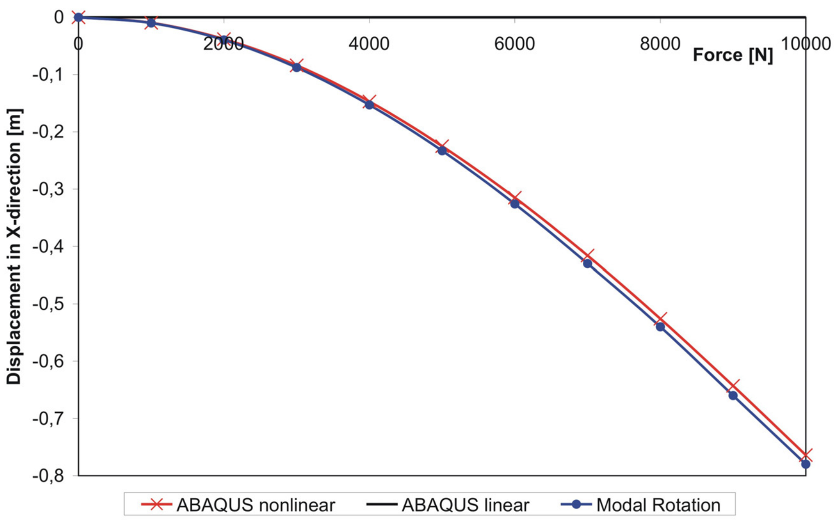

Figure 5: Displacement of the beam tip in the global X-direction

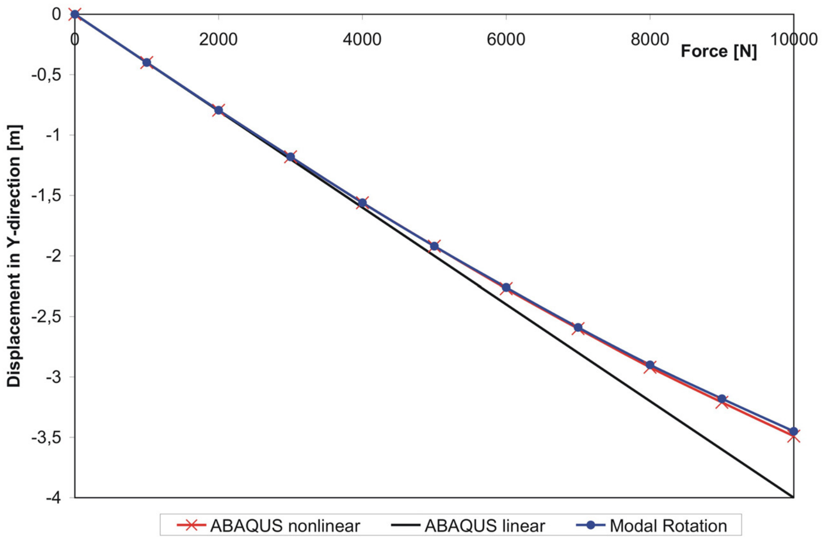

Figure 6: Displacement of the beam tip in the global Y-direction

5.2 Rear car axle

Some typical fields of application of MBS dynamics in car industry are suspension kinematics, handling performance, ride comfort, durability, etc. The increasing number of needed simulations put emphasis onto numerical efficiency, besides the classical requirement for high level of accuracy.

Figure 7 depicts the geometry of the considered rear car axle. A great part of the structure belongs to the class of thin-walled structures. Accordingly, the full FEM model (courtesy of Volkswagen AG) contains approximately 44 000 linear shell elements and 5 000 solids and over 300 000 DOF. The deformational behaviour of the axle demands consideration of geometrically nonlinear effects in order to reach the required degree of simulation accuracy. For a single static geometrically nonlinear computation based on the full FEM model, an 'average PC configuration' requires time that may even exceed an hour depending on required number of increments and boundary conditions. On the other hand, the time necessary for modal space based computation is measured in seconds, but the price to be paid is deterioration in accuracy.

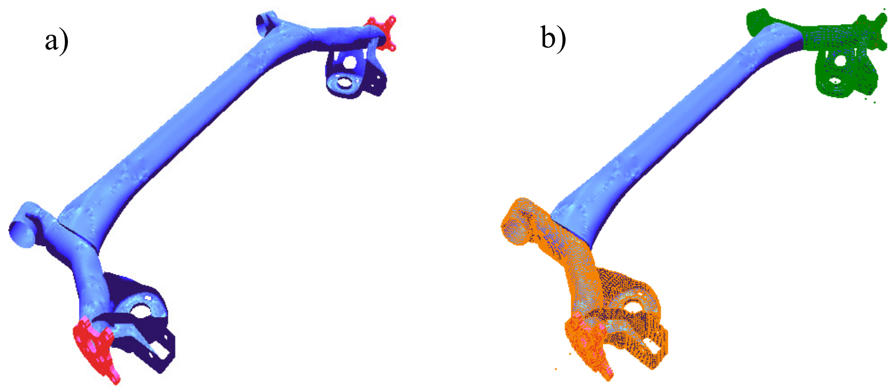

Figure 7: Rear car axle: a) CAD model; b) sub-domains for rotation

The rear car axle is a viable candidate structure for application of the proposed method. The crank arms of the axle may perform an average rotation of up to 15° over the course of the axle's deformation, while the deformation of the arms themselves remains rather small. In other words, the arms mainly perform a rigid-body rotation during deformation. Therefore, they are chosen as structural sub-domains (Figure 7b), for which the displacement rotation is to be performed. In order to determine the rotation, for each of the arms four characteristic points are chosen as a 3D structure is handled here. One of them is at the axle bushing, where the axle kinematic boundary conditions are defined, and the remaining three are coupled to the axle flange by means of adequate constraints. The four points determine tetrahedrons in both original and deformed configurations. There is a unique matrix that maps the tetrahedron from the original configuration into the same tetrahedron in the deformed configuration. Polar decomposition of the matrix yields the rotation matrix.

Considering deformational behaviour of a rear car axle, the quantities that are quite often of special interest for the car industry are vertical wheel displacement (suspension travel) and toe angle. Those quantities influence the turning curve of the car. If car dynamics is simulated in an MBS program, the rear car axle is often considered as a flexibly body due to this influence. Hence, the accuracy of computing the aforementioned quantities is of significance for the overall simulation.

For two specially selected representative load cases (defined by Volkswagen AG), those quantities are computed with different approaches and represented against each other on diagrams in Figures 8 and 9. Load case 1 is a vertical force of 1 kN acting upon the wheel, while load case 2 is a side force of 7 kN, both of which are given in the figures together with the boundary conditions. The reference solutions are the linear and geometrically nonlinear one from ABAQUS, computed using the full FEM model. The diagrams also contain the solutions obtained in ADAMS. The ADAMS solution is based on the Craig-Bampton modes, hence a combination of both normal and static modes, and incorporates a single rigid-body rotation (determined by the floating reference frame). One may notice that such a combination yields a relatively good agreement with the geometrically nonlinear solutions from ABAQUS in the considered cases. The solution obtained by the proposed method (denoted by Modal_Rotation) is based on 20 normal eigenmodes. As already emphasized, the single rigid-body rotation is not included. Displacements of the crank arms are regarded separately. This solution also represents a relatively good approximation of the geometrically nonlinear full FEM solution.

In certain cases of deformational behaviour, the average rigid-body rotation would be negligible. One may think of the case in which the crank arms are moving in the opposite directions. In such a case, the solution yielded by the approach based

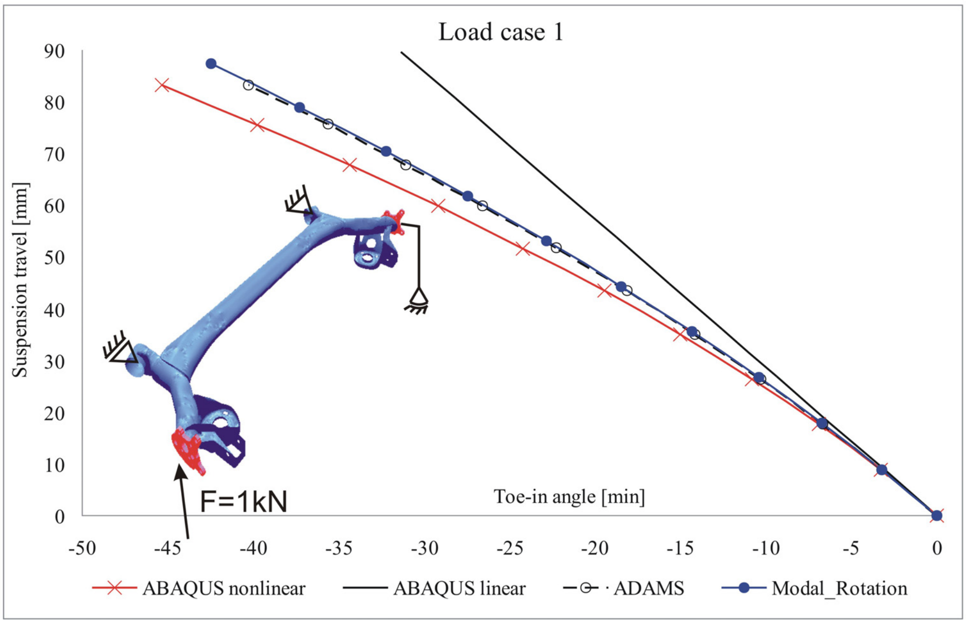

Figure 8: Vertical wheel displacement vs. toe-in angle for load case 1

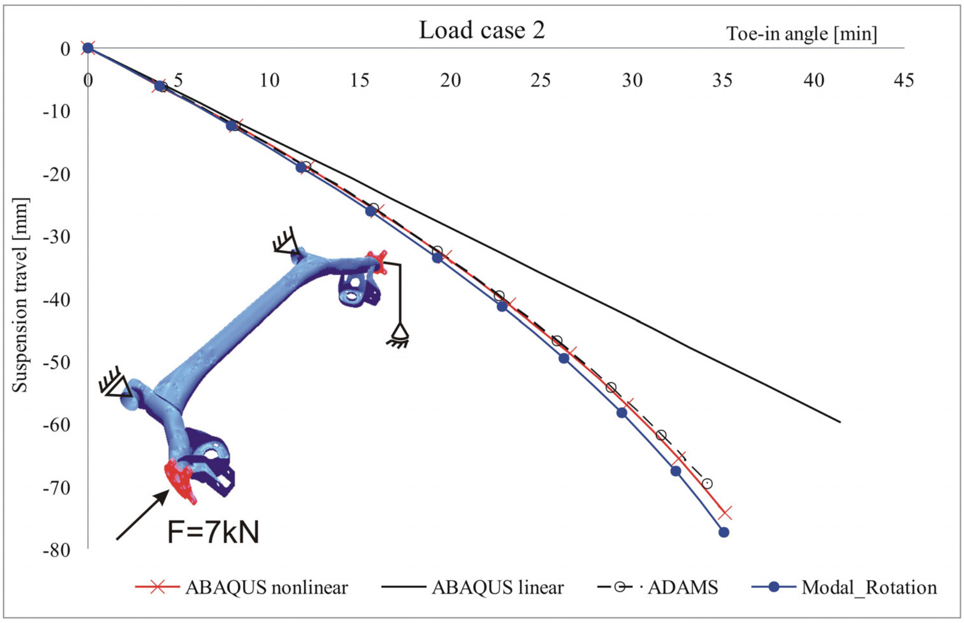

Figure 9: Vertical wheel displacement vs. toe-in angle for load case 2

on the floating reference frame would suffer on accuracy and it would be of very similar quality offered by the solution obtained with the linear computation based on the full FEM model. On the other hand, the solution based on partial rotation of displacements would still yield a reasonably good approximation of the geometrically nonlinear full FEM solution.

6 Conclusions

Integration of flexible bodies in MBS dynamics is a key requirement for successful simulation of a number of modern mechanisms, especially those involving thin-walled structures. Not only do such structures exhibit non-negligible elastic behaviour, but their elastic behaviour may also feature significant geometrically nonlinear effects. Consideration of geometrically nonlinear effects is supposed to offer improved simulation accuracy but with the important requirement of retaining the high numerical efficiency of the modal space based solution. Hence, the paper considers methods of accounting for geometrical nonlinearities of moderately large deformations computed in modal space.

The already existing solutions that use the geometric stiffness matrix are addressed in short. A new method is proposed to account for moderately large local rigid-body rotations. The method has been tested for beam-like structures using normal eigenmodes (computed with the predefined boundary conditions) and without the floating reference frame. The considered rear car axle is a representative of more complex structures that contain several sub-domains which can perform moderately large rigid-body rotations with respect to each other. The considered examples demonstrated a noticeable improvement offered by the method compared to the linear solution. It was seen that a formulation with the floating reference frame may also yield a good approximation for geometrically nonlinear behaviour, but the proposed method offers a greater flexibility, since the rotation of displacements can be performed for each sub-domain separately, whereas the floating reference frame uses a single average rigid-body rotation.

The proposed approach needs further testing of various aspects. The first one would be its applicability to deformational behaviour of the structures that cannot be described as beam-like. A further aspect is the division of flexible bodies into sub-domains. Whereas in certain cases this might be an obvious task (as in the case of the rear car axle), for many structures this can be an ambiguous task. Similarly, the ambiguity of computation of the average rigid-body rotation of sub-domains needs to be given attention in order to come up with either a general solution or, at least, recommendations on how to proceed with different classes of structures. The approach should also be tested in combination with the floating reference frame. While it was possible to implement the geometric stiffness matrix approach into the software package ADAMS and test it in this manner with the MBS system [6], the same could not be done with the approach proposed here due to limitations of the user-defined subroutines in ADAMS.

It should be appreciated that this approach can be combined with the method that incorporates the geometric stiffness matrix. In that manner, both the stress stiffening effects and moderate local rigid-body rotations could be taken into account in modal space based solutions. Of course, any of these approaches has to be used with great caution as their applicability strongly depends on the structure itself, applied boundary conditions, amount of caused deformation, etc.

Acknowledgment

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