



## **Assessment of the Stress-Strain State of Earth Dams**

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### **Abstract**

Mathematical statement, methods and algorithms of solution of the problem of assessment of stress-strain state and dynamic behavior of earth structures are considered in this paper which account for structural features, elastic-plastic and non-linear-moisturing properties of soil under static and dynamic effects. The problem is considered within the limits of plane deformation. The stress-strain state and dynamic behaviour of certain structures under different effects are studied. Some results revealed in the investigation have considerable practical importance.

**Keywords:** earth dams, non-linear, moisturing, elastic-plastic characteristics, bi-linear diagram, accelerograms of earthquakes.

## **1 Introduction**

If, to describe the equation of state of earth ambient, the hypothesis of energy form-modification is used and elastic-plastic deformation of soil is described by bi-linear [1] diagram  $\sigma_i = \sigma_i(\varepsilon_i)$ , then the solution of the problem by the method of variable parameters of elasticity becomes simple. According to this hypothesis the transfer from elastic state into plastic one in discussed particle of ambient occurs when the value of intensity of normal stresses  $\sigma_i$  in this point reaches the limit of yielding  $\sigma_T$ . Hence the state of earth ambient in different points is described differently [2-4].

Basic model of soil deformation [5] and its improved options, proposed in [6-9], gives an opportunity to take into consideration not only non-linear law of soil deformation under compression, but also the change of strength characteristics of water-saturated soils with account of structural destruction. Here the functions of modulus of compression and tension with several dimensionless coefficients, which

characterize the degree of changes of modulus of compression and tension, are determined from results of experiments for certain soils [10]. Models [2-9] allow the study of the stress-strain state of structures built of soils with different moisture content.

## 2 Mathematical Statement of the Problem and the Methods of Solution

Plane-deformation state of heterogeneous structure (Figure 1) is considered; with volume  $V = V_1 + V_2 + V_3$ , located on rigid foundation  $\Sigma_u$  under the effect of mass forces  $\vec{f}$  and hydrostatic water pressure  $\vec{p}_c$ . The surfaces  $(\Sigma_1 - S_p)$ ,  $\Sigma_2$ ,  $\Sigma_3$  are free from stresses; in foundation (on the surface  $\Sigma_u$ ) a kinematic effect  $\vec{u}_0(x_1, x_2, t)$  is applied, and on  $S_p$  - hydrostatic water pressure  $p_c(x_1, x_2)$ .

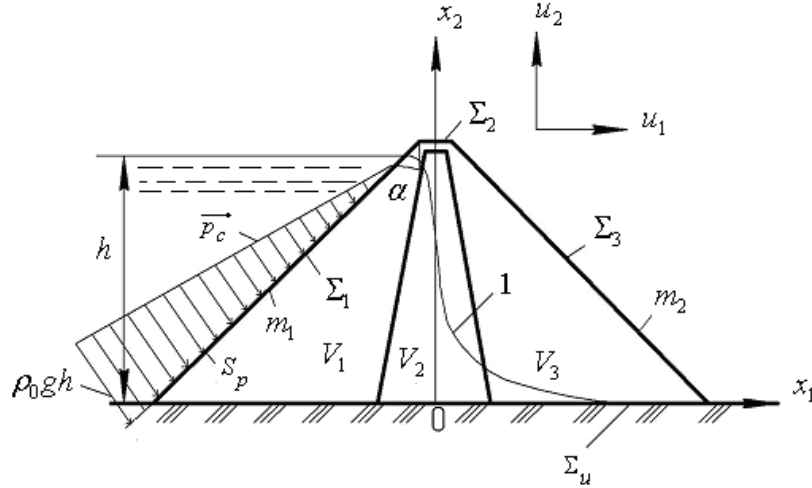


Figure 1: Model of heterogeneous structure on rigid foundation

To simulate the process of deformation of a structure (Figure 1) under different effects, Lagrange-D'Alembert variational equation of dynamics is used

$$\begin{aligned}
 & - \int_{V_1} \sigma_{ij} \delta \varepsilon_{ij} dV - \int_{V_2} \sigma_{ij} \delta \varepsilon_{ij} dV - \int_{V_3} \sigma_{ij} \delta \varepsilon_{ij} dV - \\
 & - \int_{V_1} \rho_1 \ddot{u} \delta \bar{u} dV - \int_{V_2} \rho_2 \ddot{u} \delta \bar{u} dV - \int_{V_3} \rho_3 \ddot{u} \delta \bar{u} dV + \\
 & + \int_{S_p} \vec{p}_c \delta \bar{u} dS + \int_V \vec{f} \delta \bar{u} dV = 0
 \end{aligned} \tag{1}$$

Kinematic conditions in the foundation are:

$$\vec{x} \in \Sigma_u : \quad \vec{u}_0(\vec{x}, t) = \vec{\psi}_1(t) \tag{2}$$

And initial conditions at  $t = 0$ :

$$\vec{x} \in V: \vec{u}(\vec{x}, 0) = \vec{\psi}_2(\vec{x}), \dot{\vec{u}}(\vec{x}, 0) = \vec{\psi}_3(\vec{x}) \quad (3)$$

To describe the connection between the tensors of stresses  $\sigma_{ij}$  and strains  $\varepsilon_{ij}$  in elastic case, a generalized Hook's law is used

$$\sigma_{ij} = K_n \varepsilon_{ij} \delta_{ij} + 2G_n e_{ij} \quad (4)$$

$$K_n = \frac{E_n}{3(1-2\nu_n)}, G_n = \frac{E_n}{2(1+\nu_n)} \quad (5)$$

The connection between the components of the tensors of deformation and vectors of displacements is described by Cauchy linear correlations

$$\varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (6)$$

In formulae (1)-(6) the following symbols are used:  $e_{ij} = \varepsilon_{ij} - (1/3)\theta\delta_{ij}$ ;  $\theta = \varepsilon_{ij}$ ;  $i, j = 1, 2$ ;  $\vec{u} = \{u_1, u_2\}$  - components of the vector of displacement in  $\vec{x} = \{x_1, x_2\}$  system of coordinates;  $\delta\vec{u}$ ,  $\delta\varepsilon_{ij}$  - isochronic variations of displacements and deformations;  $\rho_n$  - density of material of elements ( $V_1, V_2, V_3$ ) of discussed system;  $\vec{f}$  - vector of mass forces;  $\rho_0$  - density of water;  $g$  - acceleration due to gravity;  $\vec{p}_c$  - hydrostatic water pressure;  $\vec{\psi}_1$  - given function of time;  $\vec{\psi}_2, \vec{\psi}_3$  - given function of coordinates;  $K_n, G_n$  - volume and shear modulus of elasticity, respectively;  $E_n$  - modulus of elasticity;  $\nu_n$  - Poisson's ratio;  $\delta_{ij}$  - Kronecker's symbol;  $t$  - time. Index  $n = 1, 2, 3$  means the ratio of this parameter to  $n$ -part of structure,  $m_1, m_2$  - coefficients of laying of slopes, and the line «1» in Figure 1 shows depression curve.

So, general problem is formulated by the following way: it is necessary to determine the fields of displacements  $\vec{u}(\vec{x}, t)$  and stresses  $\sigma_{ij}(\vec{x}, t)$  in heterogeneous structure (Figure 1), that satisfy the equations (1), (4), (6) and conditions (2), (3) under the effect of  $\vec{f}$  and  $\vec{p}_c$  at any possible displacement  $\delta\vec{u}$ .

## 2.1 Determination of Stress-strain State of Structure under Static Effects

In this case the statement of the problem becomes simple, as variation equation (1) does not account inertia forces, initial conditions are absent and kinematic boundary

conditions have the form:

$$\vec{x} \in \Sigma_u : \vec{u} = 0 \quad (7)$$

### 2.1.1 Algorithm of Solution of the Problem with Account of Elastic-plastic Deformation of Soil

With account of elastic-plastic deformation of soil ambient, a hypothesis of energy form-modification is used; according to it the transfer from elastic state (4) into plastic one in a discussed point of ambient occurs when in a given point the limit of yielding is increasing, that is  $\sigma_i \geq \sigma_T$ , where  $\sigma_i$  - is an intensity of normal stresses,  $\sigma_T$  - the limit of yielding of material.

Here in formulae (5) for modulus of all-round compression and shear  $K_n, G_n$ , instead of elastic parameters  $E_n, G_n, \nu_n$ , variable parameters  $E_n^*, G_n^*, \nu_n^*$  are used, determined by the following way [2,11]:

$$E_n^* = \frac{\frac{\sigma_i^*}{\varepsilon_i}}{1 + \frac{1-2\nu_n}{3E_n} \frac{\sigma_i^*}{\varepsilon_i}}; G_n^* = \frac{\sigma_i^*}{3\varepsilon_i}; \nu_n^* = \frac{\frac{1}{2} - \frac{1-2\nu_n}{3E_n} \frac{\sigma_i^*}{\varepsilon_i}}{1 + \frac{1-2\nu_n}{3E_n} \frac{\sigma_i^*}{\varepsilon_i}} \quad (8)$$

Connection between "variable parameters of elasticity"  $E_n^*, G_n^*, \nu_n^*$  has the same form as the ones for elastic constants  $E_n, G_n, \nu_n$ , that is

$$G_n^* = \frac{E_n^*}{2(1+\nu_n^*)} \quad (9)$$

Altered physical-mechanical parameters (8) in each point of structure are determined according to deformation state reached  $\varepsilon_i$  (intensity of deformations) and corresponding intensity of normal stresses  $\sigma_i^*$  in deformation diagram  $\sigma_i^* = \sigma_i^*(\varepsilon_i)$ , which are selected from experimental data for certain soils [12, 13].

Intensity of stresses  $\sigma_i$  and strains  $\varepsilon_i$  is determined according to formula

$$\sigma_i = \frac{1}{2} \sqrt{(\sigma_{11} - \sigma_{12})^2 + (\sigma_{11} - \nu(\sigma_{11} + \sigma_{22}))^2 + (\sigma_{22} - \nu(\sigma_{11} + \sigma_{22}))^2 + 6\sigma_{12}^2}, \quad (10)$$

$$\varepsilon_i = \frac{\sqrt{2}}{2(1+\nu)} \sqrt{(\varepsilon_{11} - \varepsilon_{22})^2 + \varepsilon_{11}^2 + \varepsilon_{22}^2 + \frac{3}{2}\varepsilon_{12}^2}$$

Further, discussed problem with the use of the method of finite elements [2, 14] is reduced to non-linear algebraic system of equations of  $N$ -order

$$[K(\sigma_i, \varepsilon_i)]\{u\} = \{F\} \quad (11)$$

Here  $[K(\sigma_i, \varepsilon_i)]$  - is a matrix of rigidity of a structure;  $\{u\}$  - sought for vector of nodal displacements;  $\{F\}$  - vector of amplitudes of summed up external loads (mass forces, hydrostatic water pressure, etc.).

Coefficients of equation (11), being the elements of the matrix of rigidity  $[K(\sigma_i, \varepsilon_i)]$ , depend not only on elastic parameters, but also on stress-strain state reached in a structure.

Solution of equation (11) is carried out in several stages, in each stage the solution of algebraic equations is sought by Gauss' method.

1. In the first stage an elastic design of earth structure, being in equilibrium under the effect of applied load, is carried out. In this case the matrix of coefficients of equation (11) is a matrix of rigidity of elastic problem and does not depend on stress-strain state, that is on  $\sigma_i$  and  $\varepsilon_i$ .

2. In the second stage of design stress-strain state of structure is analysed in all finite elements. If, in some finite elements the intensity of stresses  $\sigma_i$  exceeds the limit of yielding  $\sigma_T$ , determined from experiments for certain materials, then for these elements with formulae (8) new parameters of elasticity  $E_n^*, G_n^*, \nu_n^*$  are determined; new matrices of rigidity are built for them and a general matrix for the whole structure  $[K(\sigma_i, \varepsilon_i)]$  is written. The system of equations obtained (11) is solved by Gauss' method, stress-strain state of structure is determined and further is studied to reveal its limit state.

3. The process goes on till the convergence of consecutiveness  $\sigma_i$  for the whole structure within the limits of given accuracy is reached. Described method presents the method of variable parameters of elasticity [2, 11, 13] and gives quick convergence of solution after 3–6 iterations.

### 2.1.2 Algorithm of Solution of Problem with Account of Non-linear and Moisturing Properties of Soil

When water reservoir is filled with water there occurs water filtration through the body of a dam; as a result the soil, laying under depression curve – 1, becomes water-saturated one with different physical-mechanical properties of soil, that determine stress-strain state of a structure.

In this case to solve discussed problem with account of non-linear and moisture characteristics of soil a model is used; this model accounts non-linear law of volume deformation with account of structural destruction and moisture content of soil, determined by dependences [5-9]

$$P = K_n(I_s, I_w) \cdot \theta \quad (12)$$

Here  $P$  - is a total pressure;  $\theta$  - volume deformation;  $K_n(I_S, I_W)$  - volume modulus of compression, which is the function of parameters of structural alteration of soil under compression  $I_S$  and wetting  $I_W$ , determined by formulae

$$K_n(I) = K_{nS} \exp(\alpha_1(1-I)) \quad (13)$$

The parameter of structural alteration of soil under compression and wetting  $I \in [0,1]$  is determined by the sum

$$I = I_S + I_W \quad (14)$$

Here  $I_S$  - is a parameter, characterizing structural change of soil under compressive load:

$$I_S = \theta / \theta_* \quad (15)$$

$\theta_*$  - is a value of volume deformation at which structural pattern of soil is subjected to complete destruction;  $I_W$  - characterizes structural change of soil at wetting:

$$I_W = W / W_* \quad (16)$$

$W_*$  - is the value of moisture content, at which the skeleton of soil completely loses the strength; it is determined from experiments [10, 15].

In (13)  $K_{nS}$  - is a volume modulus of soil compression; its state corresponds to the case  $\theta = \theta_*$  and  $W = W_*$ ;  $\alpha_1$  - is a dimensionless coefficient, characterizing the degree of change of volume modulus of compression  $K_n$  at compression and wetting, determined from tests.

To solve this problem a modified method of variable parameters of elasticity is used; it considers the method, proposed in [3,6], where the law of deformation (12) connects total pressure in soil  $P$  with volume deformation  $\theta$ .

Here expressions, that determine the value of variable parameters of elasticity, include variable modulus of compression  $K_n(I_S, I_W)$ , that depends on the degree of destruction  $I_S$  and moisture content of soil  $I_W$ .

In this case the expression for variable parameters will be determined from [2]:

$$E_n^* = \frac{\frac{\sigma_i}{\varepsilon_i}}{1 + \frac{\sigma_i}{\varepsilon_i} \cdot \frac{1}{9K_n(I_S, I_W)}}; \quad G_n^* = \frac{\sigma_i}{3\varepsilon_i}; \quad \nu_n^* = \frac{\frac{1}{2} - \frac{\sigma_i}{\varepsilon_i} \cdot \frac{1}{9K_n(I_S, I_W)}}{1 + \frac{1}{9K_n(I_S, I_W)}} \quad (17)$$

Here elastic correlations (9) will be true for variable parameters of elasticity  $E_n^*, G_n^*, \nu_n^*$ .

That is why worked out method may be also called the method of variable parameters of elasticity, that differs from well-known method [11, 13] by design formulae.

In this case discussed variation problem of determination of stress-strain state of structure with account of non-linear law of volume deformation and wetting of soils (12)-(17) by the method of finite elements is reduced to the system of non-linear algebraic equations of  $N$ -order:

$$[K(P_i, \theta_i)]\{u\} = \{F\} \quad (18)$$

here  $[K(P_i, \theta_i)]$  - is a matrix of rigidity of a structure, determined by modified method of variable parameters of elasticity;  $\{u\}$  - sought for vector of nodal displacements;  $\{F\}$  - vector of amplitudes of summed up external loads (mass forces, hydrostatic pressure of water, etc.).

Algorithm of realization of this method employs iteration procedure.

1. On each step of iteration by the method of finite elements with relationships (12)-(17) a system of equations (18) is written, and then solved by Gauss' method.

2. Further, for each finite element a stress-strain state is determined: tensors of stresses  $\sigma_{ij}$ , strains  $\varepsilon_{ij}$ , their intensity ( $\sigma_i$  and  $\varepsilon_i$ ), total pressure  $P$  and volume deformation  $\theta$ .

3. The value of these parameters define new values of variable parameters of elasticity (17), which further are used in formulation and solution of the system of equations (18).

4. The process is repeated till the difference between two consecutive values of  $P$  in each finite element reaches a given accuracy.

As a zero approximation, an initial value of modulus of compression at zero deformation and wetting is used [15]:

$$K_n = K_{ns} \exp(\alpha) \quad (19)$$

## 2.2 Determination of Dynamic Behaviour of a Structure under Static Loads and Kinematic Effect

To assess dynamic behavior of heterogeneous structure a variation equation is solved (1), (4)-(6) under kinematic effects (2), with initial conditions (3).

Physical relationships used here (4) depend on the character of soil deformation.

### 2.2.1 Algorithm of Solution of the Problem with Account of Elastic-plastic Deformation of Soil

At elastic-plastic deformation of soil a model is used, in which the equation of state of earth ambient, using the hypothesis of energy of form-modification, are described in [12] by the following equations (Figure 2):

1. Elastic stage (loading and unloading)  $\sigma_i \leq \sigma_T^0$ ,

$$\sigma_i = E\varepsilon_i \quad (20)$$

At this stage the relations of generalized Hook's law are used: where  $\sigma_i$ ,  $\varepsilon_i$  - is an intensity of stresses and strains, determined by formula (10).

2. Plastic stage (loading) is characterized by relationships  $\sigma_i > \sigma_T^0$ ;  $\frac{d\varepsilon_i}{dt} > 0$ ,

$$\sigma_i = E_{pl}\varepsilon_i - E_{pl}\varepsilon_T^0 + \sigma_T^0 \quad (21)$$

Elastic stage begins at  $\frac{d\varepsilon_i}{dt} < 0$  (unloading and loading)  $\sigma_i \leq \sigma_T^0$ ;

$$\sigma_i = E\varepsilon_i - E\varepsilon_T' + \sigma_T' \quad (22)$$

3. Plastic stage (loading)  $\sigma_i > \sigma_T$ ;  $\frac{d\varepsilon_i}{dt} > 0$ ,

$$\sigma_i = E_{pl}\varepsilon_i - E_{pl}\varepsilon_T^0 + \sigma_T, \quad (23)$$

here  $E$ ,  $E_{pl}$  - are modulus of elasticity or tangents of the angle of decline of elastic and plastic parts of a diagram;  $\sigma_T^0$ ,  $\varepsilon_T^0$  - are initial values of the limit of elasticity;  $\sigma_T'$  - the value of intensity of stresses, reached at the moment of unloading, that is new value of the limit of elasticity;  $\varepsilon_T'$  - corresponding value of intensity of deformation.

Solution of discussed problems is sought in the form:

$$\vec{u}(\vec{x}, t) = \vec{u}_0(\vec{x}, t) + \vec{u}^*(\vec{x}, t), \quad (24)$$

where  $\vec{u}^*(\vec{x}, t)$  - is a vector of sought for displacements;  $\vec{u}_0(\vec{x}, t)$  - kinematic effect at the foundation.

Lagrange-D'Alembert variation equation of dynamics (1) by the method of finite elements [2, 14] is reduced to non-linear system of differential equations of higher order in the form

$$[M]\{\ddot{u}(t)\} + [C]\{\dot{u}(t)\} + [K^*(\sigma_i, \varepsilon_i)]\{u(t)\} = \{F\} + \{f(t)\}, \quad (25)$$

With initial conditions

$$\{u(0)\} = \{u_0\}, \quad \{\dot{u}(0)\} = \{v_0\} \quad (26)$$



here  $[M]$  - is a matrix of masses of discussed system;  $[C]$  - matrix of dissipative forces (is they were taken into consideration);  $[K^*(\sigma_i, \varepsilon_i)]$  - general matrix of rigidity, depending on reached stress-strain state of a structure, according to diagram of deformation (Figure 2);  $\{u(t)\}$  - sought for vector of displacements;  $\{f(t)\}$  - vector of external load from kinematic effect;  $\{F\}$  - summed up vector of external loads (mass forces, hydrostatic pressure of water, etc.)

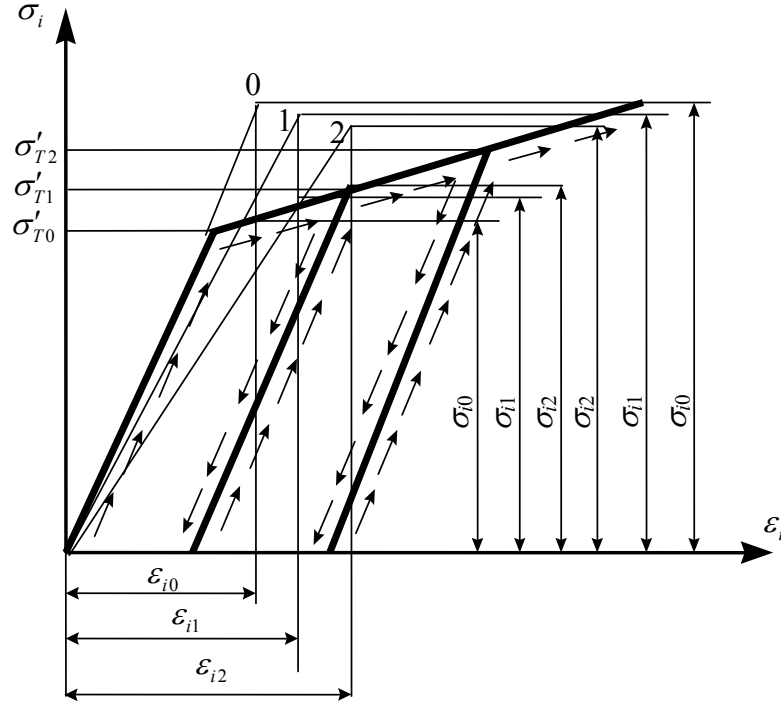


Figure 2: Diagram of deformation and the procedure of the method of variable parameters of elasticity

Discussed mechanical problem, accounting elastic-plastic deformation of material of a structure, formally (as well as in elastic problem) is reduced to solvable system of differential equations (25) with initial conditions (26). However, to form general matrix of rigidity  $[K^*(\sigma_i, \varepsilon_i)]$ , which depends on stress-strain state of a structure, the method of variable parameters of elasticity is used on each step (Figure 2).

1. Solution of dynamic problem with account of elastic-plastic deformation consists in the following.

At initial moment  $t = t_0$  the values of elastic parameters ( $E_n, G_n, \nu_n$ ) are taken as an initial parameters to build a matrix of rigidity and using (4)-(5) equations (25) are formulated. Formulated system of equations (25) with initial conditions (26) is solved by Newmark's method [2, 14] for the one step in time, that is, the solution is obtained only for the moment  $t_1 = t_0 + \Delta t$  ( $\Delta t$  - constant step of integration).

2. According to displacements obtained,  $(u_i)_{t=t_1}$  strains  $(\varepsilon_{ij})_{t=t_1}$ , stresses  $(\sigma_{ij})_{t=t_1}$  are determined, and hence intensity of strains  $(\varepsilon_i)_{t=t_1}$  and stresses  $(\sigma_i)_{t=t_1}$  in each point of structure.

3. For each finite element conditions (20)-(23) are checked. Satisfying condition (20) for all finite elements of a structure, the transfer to the following step  $t = t_2$  is carried out, without the change of matrix  $[K^*(\sigma_i, \varepsilon_i)]$ .

4. If for a certain element the condition (21) is satisfied, then the matrix  $[K^*(\sigma_i, \varepsilon_i)]$  is formulated anew with account of new parameters  $E_n^*, G_n^*, \nu_n^*$  in this element. For elements, satisfying condition (20), at formulation of  $[K^*(\sigma_i, \varepsilon_i)]$  initial parameters of elasticity  $(E_n, G_n, \nu_n)$  are used.

5. For the elements being at the stage of plastic deformation (satisfying conditions (21)), relationships (4) are used at formulation of matrix  $[K^*(\sigma_i, \varepsilon_i)]$  with account of parameters  $E_n^*, G_n^*, \nu_n^*$ , determined by formulae (8)-(9).

Parameters  $E_n^*, G_n^*, \nu_n^*$  are variables and depend on intensity of stresses  $(\sigma_i)_{t=t_1}$  and strains  $(\varepsilon_{ij})_{t=t_1}$ , which are the functions of coordinates and time.

6. After formulation of general matrix of rigidity  $[K^*(\sigma_i, \varepsilon_i)]$  with account of elastic-plastic deformation, an equation (25) is solved by Newmark's method with initial conditions (26). Nodal displacements obtained at the next moment of time are used to calculate  $(\varepsilon_{ij})_{t=t_2}$ ,  $(\sigma_{ij})_{t=t_2}$ ,  $(\varepsilon_i)_{t=t_2}$ ,  $(\sigma_i)_{t=t_2}$  in all finite elements of a structure and the check of conditions (20)-(23) is carried out.

7. This procedure: calculation of nodal displacements, stresses, strains, their intensity, the check of conditions (20)-(23) and appearance of new «elastic parameters», formulation of matrix  $[K^*(\sigma_i, \varepsilon_i)]$  and continuation of calculations for the next step by Newmark's method – goes on till the end of the time of the process.

8. If obtained values of intensity of stresses  $\sigma_i$  in each finite element do not satisfy given accuracy, the solutions are repeated with less step in time  $\Delta t$  till the solution with given accuracy is achieved.

### 2.2.2 Algorithm of Solution of the Problem with Account of Non-linear and Wetting Characteristics of Soil

In this case finite-element discretization of variation equation with account of (12)-(17) reduces discussed problem of determination of dynamic behavior of heterogeneous structure with account of non-linear and wetting characteristics of soil under kinematic effects (2) to the solution of non-linear system of differential equations of the type

$$[M]\{\ddot{u}(t)\} + [C]\{\dot{u}(t)\} + [K^*(P, \theta)]\{u(t)\} = \{F\} + \{f(t)\} \quad (27)$$

with initial conditions (26).

Here  $[K^*(P, \theta)]$  - is a general matrix of rigidity of a structure, which accounts non-linear and wetting characteristics of soil.

Stress-strain state of earth structures considerably depends on the change of physical-mechanical properties of soil as a result of its wetting; the higher moisture content the closer the state of soil to plastic one. That is why to solve this problem variable parameters of elasticity are used similar to the procedure, described above.

### **3 Study of Stress-strain State of Earth Dams under Static Loads with Account of Elastic-plastic and Wetting Characteristics of Soil**

With worked out algorithm an assessment of stress-strain state of two acting earth dams (Sokh and Ghissarak) was carried out with account of their heterogeneity, structural features, elastic-plastic deformation and wetting characteristics of soil under mass forces and hydrostatic pressure of water.

The height of Sokh dam is  $H = 87.3\text{m}$ ; lying of slopes of upper prism  $m_1 = 2.5$ , lower prism,  $m_2 = 2.2$ . The structure presents heterogeneous structure, consisting of a kernel (loam), retaining prisms (gravel-crashed stone soil), additional loading (broken and large-block stone)

Ghissarak dam has the height  $H = 138.5\text{m}$ , lying of upper slope is  $m_1 = 2.2$ , lower one  $m_2 = 1.9$ . The structure also presents heterogeneous structure, consisting of a kernel (loam), side retaining prisms (filled with rock mass from lime-stone quarry).

#### **3.1 Assessment of Stress-strain State of Dams with Account of Elastic-plastic Properties of Soil**

Elastic-plastic properties of soil are accounted by bi-linear diagram of deformation  $\sigma_i = f(\varepsilon_i)$  with the degree of strengthening  $\bar{\lambda} = (1 - E_n / E) = 0.75$ , that is an angle of plastic part  $E_n$  is supposed four times less than the decline of elastic part  $E / E_n = E / 4$  [1]. The limit of yielding is taken according to [16], for the material of a kernel (loam) -  $\sigma_T = 0.3\text{MPa}$ , and for material of retaining prisms within the limits  $\sigma_T = 0.45 - 0.50\text{MPa}$ . Here  $E$ ,  $E_n$  - are declines of elastic and plastic parts of diagram, respectively.

In Figure 3 the lines of distribution of the levels of intensity of normal stresses  $\sigma_i$  in the section of heterogeneous Sokh dam, obtained with account of structural features and elastic-plastic deformation of soil under own weight are shown. Comparison of these results with the ones, obtained for the same dam in elastic statement shows that an account of elastic-plastic properties of soil of the prism and kernel leads to qualitative and quantitative change of stress state of a dam: down to  $\approx 20\%$  drops the intensity of stresses  $\sigma_i$ , vertical stresses in a kernel; arc effect is

strengthening in the zone of a kernel. The reason of this is the difference of deformation properties and the limits of yielding of soils of a kernel and retaining prisms. Here in upper part of a dam horizontal stresses are increasing up to 0.05MPa, as a result the change of the profile of a dam becomes possible. An increase of tangent stresses up to 0.05-0.06MPa may also lead to the change of profile and to decrease the coefficient of safety factor in slope zones.



Figure 3: Lines of distribution of the levels of intensity of stresses  $\sigma_i$  in the section of Sokh dam

In Figure 4 the lines of distribution of the levels of intensity of stresses  $\sigma_i$  in heterogeneous Ghissar dam with account of structural features and elastic-plastic properties of soil under own weight are shown.

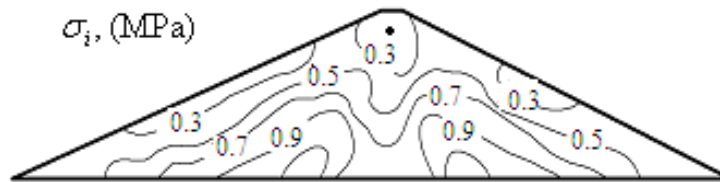


Figure 4: Lines of distribution of the levels of intensity of stresses  $\sigma_i$  in the section of Ghissarak dam

Comparison of obtained results (Figure 4) with existing data shows that in Ghissarak dam an account of elastic-plastic deformation of soil leads to re-distribution of stress state in the following way: down to  $\approx 15\%$  drops the intensity of stresses  $\sigma_i$  and vertical stresses in a kernel; arc effect is strengthening; in upper and lower slope zones tangent stresses grow up to  $\approx 50\%$ ; it leads to the decrease of coefficient of safety factor near the slopes.

Besides, results obtained show that with a decrease of incline of plastic part in deformation diagram  $\sigma_i = f(\varepsilon_i)$  of soil, the difference in results of solution of elastic-plastic problem will be considerable comparing with solution of elastic problem. So a substantiated approach to the choice of the law of deformation of soil is necessary.

An analysis of results obtained for the dams of different geometry and height shows that an account of elastic-plastic deformation of material of retaining prisms and kernels differently influences their stress-strain state. The influence of elastic-plastic properties of soil on stress-strain state of a dam becomes more profound with the height of a dam. Arc effect is strengthening due to the drop of intensity of

stresses  $\sigma_i$  down to  $\approx 18\%$ , and vertical stresses – down to  $\approx 20\%$ . Tangent stresses are considerably increasing in upper and lower retaining prisms - up to  $\approx 40-55\%$ . The greatest difference is observed near slopes, as a result landslides occur and shear deformations develop. In the zone of a kernel tangent stresses are insignificant.

Theoretical investigations show that in projecting of high and averagely high dams it is necessary to carry out design with account of structural features and real elastic-plastic properties of soil.

### 3.2 Assessment of Stress-strain State of Dams with Account of Non-linear and Wetting Properties of Soil

In this chapter stress-strain state of earth dams is evaluated by worked out algorithm with account of structural features of structures at optimal moisture content of soil under different kinematic effects.

In correspondence with data given in [17], the value of optimal moisture content at compaction is taken within the following limits: for sandstone  $\approx 7-10\%$ , for sand loam  $\approx 9-15\%$ , for loam  $\approx 12-20\%$ . At compaction of gravel, alluvial soils and rock mass the value of optimal wetting may be considered equal to moisture content of soil in natural state that is  $\approx 5-12\%$ .

The values of parameters used in design of the model of soil were taken according to [6, 18]: for loamy soil  $K_{ns} = 28 - 30$  MPa,  $\alpha = 2.5$ ,  $\theta_* = 0.0015$ ; for materials of the prism  $K_{ns} = 38 - 42$  MPa,  $\alpha = 2.0$ ,  $\theta_* = 0.002$ .

In Figure 5 the lines of distribution of the levels of intensity of normal stresses in section of heterogeneous Sokh dam with account of elastic-plastic properties and water-saturation of soil are shown. The following values of optimal wetting were used in calculations: for soils of a prism  $W = 10\%$ , for kernel  $W = 16\%$ . Comparison of results, obtained in elastic statement, shows that an account of elastic-plastic properties and optimal wetting of soil leads to the change of character of stress-strain state of a dam.

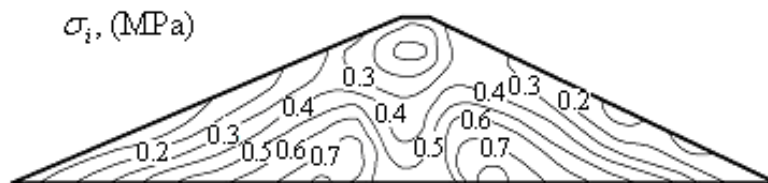


Figure 5: Lines of distribution of the levels of intensity of stresses  $\sigma_i$  in section of Sokh dam with account of elastic-plastic and wetting properties of soil

These factors lead to the strengthening of arc effect in a kernel, specific for intensity of stresses  $\sigma_i$  and vertical normal stress. The values of intensity of stresses  $\sigma_i$  are decreasing in the kernel down to  $\approx 25\%$  and vertical stresses – down to  $\approx 20\%$ . Tangent stresses reach the value of 0.06MPa.

Results obtained show, that in bi-linear model soil deformation occurs when an intensity of stresses  $\sigma_i$  reaches the limit of yielding, but according to the model offered in [15], non-linear elastic-plastic properties of soil are revealed at the beginning of loading. That is why using the second model one may account residual deformations even in those parts of a dam, where the values of intensity of stresses do not exceed the limit of yielding  $\sigma_T$ . Principal advantage of this model is the possibility to account moisture content and to use real parameters  $(K_{nS}, \alpha, \theta_*)$  of soil in concrete calculations.

This gives an opportunity to solve a number of new engineering problems, connected with soils, in particular, when studying stress-strain state of earth dams. Results obtained from each of these two models show similar pattern of stress-strain state of earth dams, the difference being about  $\approx 7\%$  for certain components of stresses.

Based on obtained results we may draw a conclusion on necessity of account of wetting of soil, as it significantly influences stress-strain state of a dam, strengthening arc effect in the kernel, specific for intensity of stresses  $\sigma_i$  and vertical normal stresses. An intensity of stresses near foundation and in prisms is  $\approx 10\%$  less, horizontal stresses in lower part of a dam are  $\approx 8\%$  greater and tangent stresses in a kernel are 2 times greater. The difference in pointed out stresses depend on the height of a dam: the higher the dam the greater the difference.

Decrease of  $\sigma_i$  and increase of horizontal stresses in lower part near foundation leads to lessening of resistance of structures to shear horizontal forces, and an increase of tangent stresses in a kernel and near upper slope – to formation of cracks and occurrence of landslides.

## **4 Study of Dynamic Behaviour of Earth Dams with Account of Elastic-plastic and Wetting Properties of Soil under Real Seismic Effect**

In this chapter unsteady forced vibrations are studied and an assessment of stress-strain state of two dams is given: heterogeneous dam with height  $H=185\text{m}$ , coefficients of lying of slopes  $m_1=2.0$  and  $m_2=1.9$ , a kernel of loam and side retaining prisms of rock mass and Ghissarak dam with account of elastic-plastic and wetting properties of soil of structure under real seismic effects. The records of registered accelerograms of Gazli earthquake [19] were used as an external effect.

### **4.1 An Assessment of Dynamic Behaviour of Dams with Account of Elastic-plastic Properties of Soil**

To account elastic-plastic properties of soil the parameters given in chapter 3.1 were used. Figure 6 shows isolines of maximal values, obtained by worked out algorithm

(during studied period of a process), of intensity  $\sigma_i$  and normal stresses in a section of heterogeneous Ghissarak dam under the effect of the record of accelerogram of Gazli earthquake with account of elastic-plastic properties of soil.

An analysis of results shows, that under high frequency intensive effect (accelerogram of Gazli earthquake), the character of oscillations of a structure with low frequency specter of natural vibrations have three distinctive stages: initial one (approximately 2 sek), when the amplitude of oscillations, deformation and stresses are small, transition stage (approximately 6 sek), when oscillations of a structure occur, and a stage of free oscillations with reached amplitude and the frequency of natural oscillations. High frequency character of the effect does not allow to completely reveal residual deformations and in spite of high intensity of accelerogram, causing great stresses in the body of observed dams, their calculations with account of elastic-plastic deformation give rather true notion of elastic design.

Under the effect of accelerogram of Gazli earthquake in the section of a dam with height  $H=185\text{m}$  maximal intensity of stresses reaches  $1.2\text{MPa}$ . Here vertical stresses are 2-3 times greater than horizontal ones. An account of elastic-plastic deformation of material leads to decrease of intensity of stresses and increase of tangent stresses in the section of a dam. Arc effect in a kernel, revealed in elastic statement, is increasing.

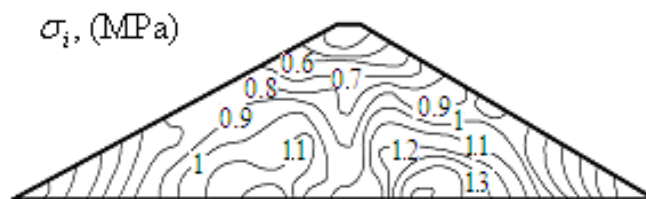


Figure 6: Isolines of maximal levels of intensity of stresses in section of Ghissarak dam under dynamic effect with account of elastic-plastic properties of soil

An analysis of results of study of unsteady forced oscillations [20, 21] of earth dams under real seismic effects (accelerograms of Gazli earthquake) shows that stress-strain state of elastic-plastic structure depends not only on maximal value of acceleration, but, in a greater degree, on its frequency spectrum and duration of oscillations. An account of elastic-plastic deformation increases the period of oscillations and with time causes significant residual deformations in a dam, mainly in its lower part. Strengthening of arc effect in the zone of a kernel, specific for vertical stresses, is a result of difference of deformation properties and limits of yielding of soils of a kernel and prisms.

#### 4.2 An Assessment of Dynamic Behaviour of Dams with Account of Different Moisture Content of Soil in the Kernel of a Dam

In Figures 7 and 8 isolines of distribution of maximal values of intensity of stresses  $\sigma_i$  in the section of heterogeneous dam with height  $H=185\text{m}$ , obtained during the whole process of oscillations, caused by the effect of accelerogram of Gazli

earthquake with moisture content of soil in a kernel  $W=10\%$ , (Figure 7) and  $W=20\%$  (Figure 8) are shown.

Comparison of results with different moisture content of soil in a kernel (Figures 7-8) of a dam shows that with an increase of moisture content an intensity of stresses in the kernel of a dam becomes less, while arc effect is strengthening. In a lower part of a dam an intensity of stresses  $\sigma_i$  becomes 25% less (at moisture content  $W=10\%$ ), 50% less - at  $W=20\%$  and more than 60% less - at  $W=30\%$ . In middle part of a dam the influence of moisture content is revealed by the similar way, but with less difference of compared results.

In retaining prisms, directly touching the kernel, the value of intensity of stresses, on the contrary, is increasing up to 30-40% depending on the level of moisture content. An account of moisture content leads to the growth of maximal values of tangent stresses in the zone of a kernel.



Figure 7: Isolines of maximal values of intensity in the section of a dam ( $H=185\text{m}$ ) at moisture content of soil in a kernel  $W=10\%$ )

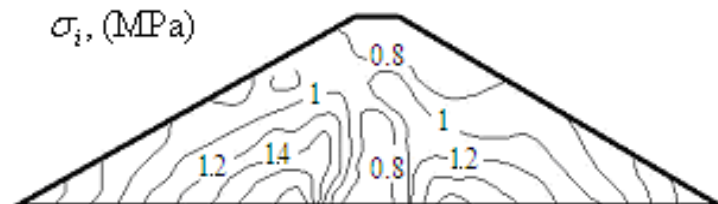


Figure 8: Isolines of maximal values of intensity in the section of a dam ( $H=185\text{m}$ ) at moisture content of soil in a kernel  $W=20\%$ )

So, wetting of a kernel significantly changes stress-strain state of a dam. With an increase of moisture content an intensity of stresses in the kernel becomes less. In resonance mode (at moisture content  $W=20\%$ ) it becomes 2 times less comparing with the case of dry kernel. Sharp change of intensity of stresses in the zone of a kernel may lead to formation of cracks both in transition zones and in the kernel itself. That is why we may draw a conclusion, that to obtain real character of stress-strain state of high dams under forced oscillations, it is necessary to account moisture content of soils, they consist of.



## 5 Conclusions

Mathematical statement and methods to assess stress-strain state and dynamic behaviour of earth dams with account of heterogeneity, structural features of structures, elastic-plastic non-linear and wetting properties of soil are worked out.

Algorithms to evaluate stress-strain state of earth structures with account of elastic-plastic and wetting characteristics of soil under static loads are elaborated.

Algorithm of solution of dynamic problems for earth structures was worked out with account of elastic-plastic and non-linear wetting properties of soil.

Stress-strain state and dynamic behaviour of certain earth dams with account of their real structural features and elastic-plastic characteristic and wetting of soil were studied.

An analysis of obtained results reveals a number of new mechanical effects, connected with the influence of elastic-plastic and wetting properties of soil on stress-strain state of structures.

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