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Parametric Geometry Generation and Automatic Aerodynamic Analysis of Aircraft Rear Fuselage and Tail Surfaces

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Abstract

In order to perform analysis driven design of aircraft components, a flexible and robust automatic geometry and analysis model generation capability is required. Parametric geometry generation using commercial CAD packages is a well established practise in industry, but the focus generally concentrates on obtaining a representative three-dimensional geometric model without specific consideration of the subsequent integration with numerical analysis methods. This paper presents a framework for the generation of the external geometry and a mid-fidelity aerodynamic analysis model of aircraft components, with particular consideration of the aft fuselage and tail surfaces of a conventional configuration. The geometry engine is based on the geometrical modulation of generic topological tubes using guiding curves and shape functions. The structured surface mesh generation for aerodynamic analysis is then performed in a bi-parametric space, taking advantage of the single patch topological representation of all the major aircraft components.

Keywords: aircraft, MDO, aerodynamics, optimisation, empennage, panel methods, NURBS, VSAERO.

1 Introduction

The evaluation of an aircraft configuration using computational analysis methods is normally based on some form of geometrical model. Traditional semi-empirical methods may be capable of producing high quality results using a reduced set of geometric parameters, for example the trapezoidal planform of a wing, as long as the geometry represented is not too different from a set of calibration models used to generate the database of responses. In these cases, the main geometric parameters are used to calculate variations of the responses of interest (weight, drag, etc…) from a reference geometry for which these data are known. The response variation may be physically based (e.g., aerodynamic induced drag reduces with aspect ratio as described by a well-known formula) or based on statistical trends, as is usually the case for weight and cost.

Clearly, the use of semi-empirically based analysis methods is limited to a restricted geometric design space for which high-fidelity results are available. These methods can only provide reliable results when a similar technology level to that contained in the knowledge database used to generate the calibration is used.

The pursuit for improved performance in aircraft design requires the consideration of unconventional configurations and novel technologies which are outside of the domain of validity of traditional semi-empirical methods.

The computational analysis methods currently available to the aircraft designer allow, in principle, to obtain high fidelity responses (aerodynamic lift and drag, stability and control parameters, weights, aeroelastic behaviour, space allocation, etc…) of any configuration. Advanced computational fluid dynamics (CFD) and finite element analysis (FEA) methods are routinely used in the detailed aircraft design phases but this is still an expensive process limited by the need to manually generate the analysis models –generally discretised meshes- which are based on a high quality and generally intricate geometric model of the aircraft.

The fundamental problem in the design of complex, high performance, systems is to reach the detailed design phase -where the expected performance can be substantiated by advanced analysis but there is reduced freedom to modify the basic definition of the product- with a robust configuration. Generally, in the conceptual development phase, where the basic geometry and technologies are being defined, the analysis driving the design generally relies on semi-empirical methods or in relatively low-fidelity numerical analysis, and this is one of the main limiting factors for the design of new configurations or the implementation of substantially novel technologies. Therefore, a key enabler for the use of advanced, high fidelity computational analysis methods during the early design phases is to have the ability to generate the detailed geometric and analysis models from a relatively reduced set of parameters.

At the lowest level of aircraft model definition, when the most basic geometric parameters are being set (span, wing area, fuselage diameter, etc…), in order to generate a high quality geometric model the use of previous design knowledge in the form of functional and geometric constraints is essential. For example, the minimum set of parameters to define a conventional slender fuselage can be its diameter and length overall. A complete geometry suitable for numerical analysis may be generated by automatically designing a "typical" nose and rear fuselage imposing tangency and curvature constraints on the guiding side and plan view curves. The resulting 3D geometry will be of sufficient quality to enable, for example, a CFD or a space allocation analysis. The aerodynamic response of this fuselage will not be of much use as many parameters are still to be defined, but these can be refined during the preliminary design process leading into the detailed design phase.

The main difficulty in implementing the previous approach is the choice of a small but sufficiently rich set of geometric parameters which will enable to represent both conventional and meaningful unconventional geometries. The geometric parameters must be then classified into hierarchical levels of dependency such that the definition of a very small number of elementary parameters and the implicit knowledge contained in the geometric and functional constraints may facilitate the complete generation of a high quality 3D model. Finally, the three-dimensional geometry must be created in a way that shall ease as much as possible the automatic generation of mid to high-fidelity numerical analysis models.

The intent is to bridge the “dimensional gap” between the conceptual and detailed definition of a complex product such as an aircraft or any of its major components and enable an “analysis driven” conceptual and preliminary design process through the use of Multidisciplinary Optimisation.

2 Geometric model generation

As discussed previously, access to a high quality geometric definition of a product is a key enabler for analysis driven design. In the case of aircraft design, including its major components (wings, fuselage, landing gear, propulsion system, tail surfaces and possibly others depending on the aircraft type and mission) one of the most important considerations is the evaluation of the aerodynamic performance of a given geometry.

Aircraft are designed from the inside-out and the design process usually starts by selecting the payload requirements which drive the “size” of the aircraft, in particular of its fuselage cross section and length of cabin and cargo compartments.

The preliminary size of the wing, landing gear and propulsion system follows and at some point the stabilising surfaces –the tail surfaces in the conventional aircraft configuration- must be defined.

The tail surfaces provide stability to the airplane as well as pitch and yaw control to satisfy handling qualities requirements. The design of the tail surfaces is driven mainly by aerodynamic considerations as these surfaces have to provide sufficient “aerodynamic stiffness” (rate of variation of lift with angle of attack or deflection of the controls) and maximum lift capability and stall angle.

There are also important geometric considerations to take into account; in particular the design of the rear fuselage, which affects the aerodynamic performance of the tails and the overall aircraft drag. The geometry of the rear fuselage may be also constrained by internal payload capacity, by rotation at take-off considerations, which influence its up-sweep angle and by the internal structural attachments of the tails, trimming mechanism, auxiliary power unit (APU) and other systems installation, as well as by interaction with ground servicing equipment and maintainability considerations.

It is clear that the overall design definition and integration of the aircraft rear fuselage and tails is a complex multidisciplinary process which can benefit substantially from the application of an analysis driven design philosophy.

The design of tail surfaces is further complicated by the fact that the stability and control requirements that they must satisfy depend on the overall definition of the complete aircraft at each design iteration. Therefore, the analysis of the tail surfaces requires access to a detailed model of the complete aircraft, in particular regarding the tail aerodynamic responses and geometric definition of the rear fuselage.

The following sections describe the approach followed to establish an analysis driven design process for aircraft rear fuselage and tail components through a computational framework referred to as MARES: Multidisciplinary Analysis of Rear-EndS

2.1 Geometry engine

The automatic generation of aircraft external surfaces from a given set of geometric parameters is a well known application of knowledge-based engineering and several examples of practical implementations are available in the literature [1,2,3,4]. However, the scope of previous approaches has generally been limited to obtaining a 3D CAD model of the aircraft or some of its components with no explicit intent to facilitate further numerical analysis [5, 6, 7].

The use of commercial CAD packages to generate external surface models has the obvious advantage that potentially all the power of the corresponding geometry engine is available to construct high quality models [8,9]. Additionally, the final product has the same format, a CAD model, used in the manual detailed design process and is the starting point for the traditional generation of CFD and FEA analysis models [10, 11,12].

There also are several drawbacks in using a generic CAD package for automatic geometry generation. As a prerequisite, the CAD system must provide some sort of programming language giving access to its geometric entities and operations. Most modern CAD programs offer some sort of "macro" language or "template" edition facility which enables to quickly set up a powerful parametric model but very often access to the geometric objects or the operations which are made available is somehow restricted. A more important limitation of some parametric models is that instantiating a quantified sets of objects (for example, deciding the number of ribs or stringers based on some distance) may not be possible, and fixing the number of entities may be the only available option.

However, in the context of enabling analysis driven design, the most important drawback of the use of CAD models is that, in general, there is no connection between the geometric entities used to define the geometry and the requirements for the construction of the discretised meshes used by the numerical analysis methods.

The geometric engine of generic CAD packages is normally based on NURBS or Bezier surface patches. To define a complex and flexible external shape parametrically, several patches, possibly with tangency and, where possible, curvature continuity constraints are used. As the number of patches increases it becomes more difficult to generate the numerical meshes required for computational analysis. Ideally, a single patch per aircraft component would be used, but this would call for a specific geometric engine since the ability to represent complex forms using traditional NURBS methods generally requires the use of various patches.

In order to facilitate the generation of the numerical meshes, required in particular for the aerodynamic and structural models, the approach followed in MARES has been to develop a numerical geometry engine suited for the definition of aircraft components. The key feature of the method is that a single geometric patch is generated for each component side. Therefore, the components are treated as “topological tubes” and the 3D geometry generation is based on the modulation of these tubes by guiding curves.

This method is effectively analogous to that used traditionally to define the lines plan of a ship hull and, although it has some limitations that will be described in what follows, provides a very efficient and flexible means of representing aircraft component geometries from the conceptual to the advanced preliminary design phase.

2.2 Fuselage parametric geometry

In general, an aircraft is composed of various major components: fuselage, wing, tails, landing gear, propulsion system and others.

The fuselage normally carries the payload (passengers and cargo) and tends to be a slender tube. In this case, the application of the “topological tube” model is easy to implement.

To define the 3D geometry of the fuselage, a set of 3 guide lines are used: crown, belly and line of maximum width.

Each of the guide lines is built by a concatenation of curves defined between control points at which tangency and curvature continuity conditions are specified. The point and tangency continuity constraints are easy to enforce and represent a necessary condition for aerodynamic smoothness.

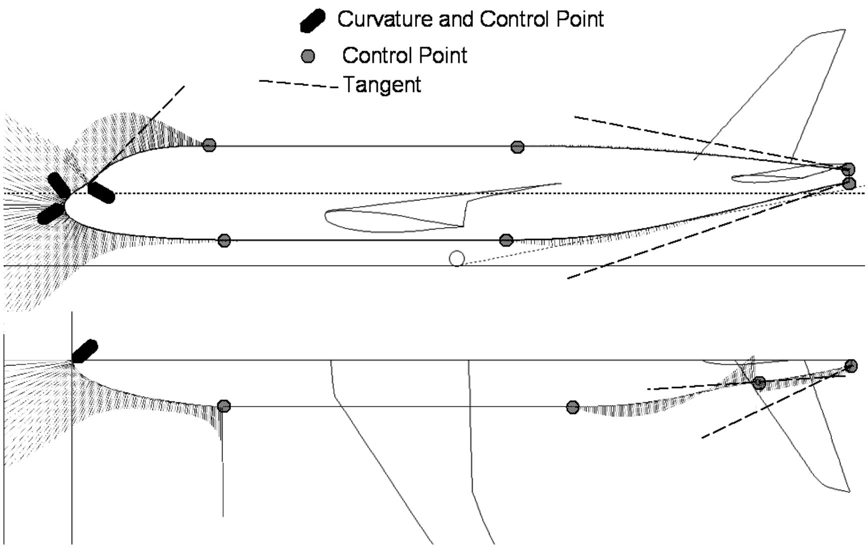

Figure 1: Fuselage definition using piecewise guide lines with internal tangent and curvature constraints

The curvature continuity is desirable as jumps in curvature may affect the development of the boundary layer. However, as each of the curves making the overall guide line is defined by low order splines, on occasions it may be difficult to obtain the desired overall shape of the guide without jumps in curvature. The designer is free to override the curvature continuity condition if required.

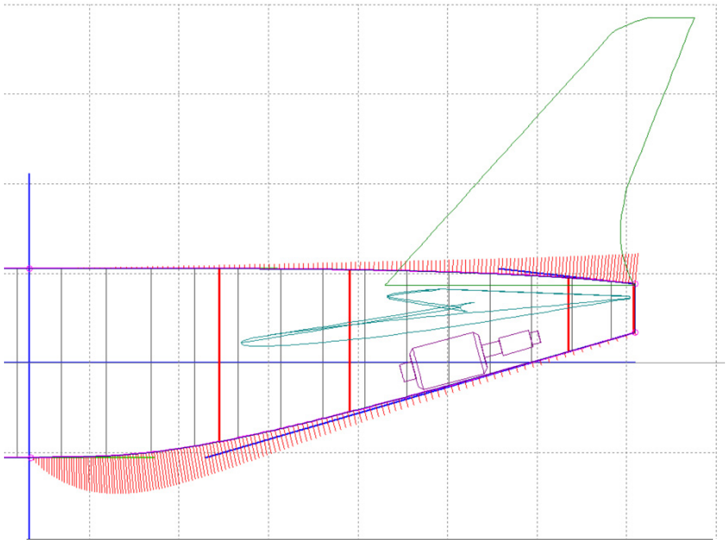

Figure 2: Side view of a rear fuselage geometric definition in MARES, showing crown and belly guiding lines and their curvature. The outlines of the auxiliary power unit (APU), vertical and horizontal tailplanes are visible.

The minimum number of curves required to represent a fuselage is driven by functional requirements. Defining a smooth nose, a cylindrical part and a rear fuselage with upsweep and taper and imposing continuity conditions, a “natural” fuselage shape in plan and side view emerges. Not all conceivable fuselages can be defined with such a small set of parameters, but in general, the ability to replicate the shape of existing conventional commercial aircraft fuselages is satisfactory.

To generate the external surface of the fuselage, its cross sections need to be defined. In MARES, this is done at master fuselage stations, where the cross section can be defined as a pure circular section or as bi-lobular, tri-lobular or tetra-lobular. The nominal basic section can then be modified by a horizontal scaling and a shape transformation which can adapt the basic section into a more square or more rhomboidal shape. With this simple shape transformation, controlled by three scalar parameters, rather exotic fuselages can be generated (figure 4).

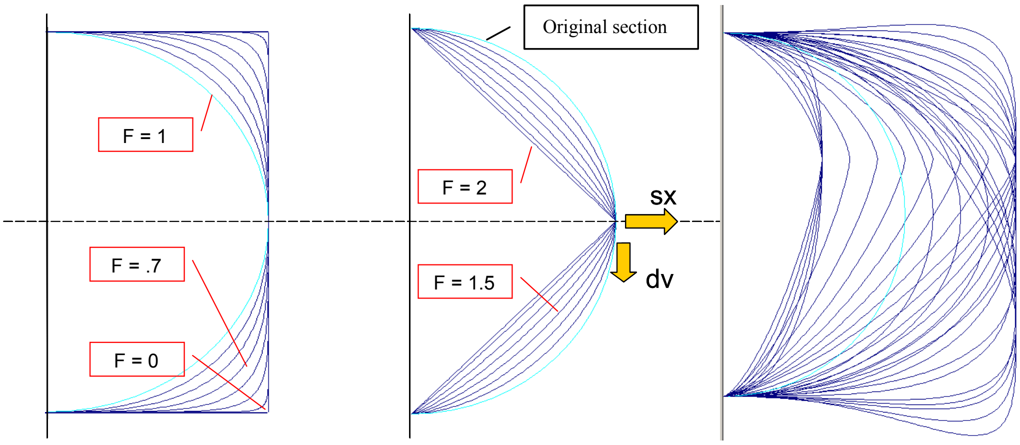

Figure 3: Modification of the basic fuselage cross sections by shape functions depending on a single parameter F

The generation of the 3D surface is performed by creating intermediate sections adapted to the curve guides. The shape of each intermediate section is obtained by means of a particularly smooth interpolation of the nominal shapes of the adjacent master station which impose first and second order continuity constraints.

Additional design freedom is provided by the use of “metaballs” to effect local perturbations on the master sections. The metaballs offset the sections according to an intensity and radial decay law which facilitates the modelling of fairings and other fuselage features in a simple way without compromising surface continuity or the need to generate additional splines.

A particularly complex design feature which consideration is mandatory for a complete definition of a rear fuselage with a conventional trimmable horizontal tailplane is the functional area. This is a surface, part of it of revolution, which enables the rotation of the horizontal tailplane around its trim axis without creating geometric gaps with the fuselage. In MARES, the functional area is created by effecting a horizontal offset of the basic fuselage surface using a specific mathematical function satisfying the aerodynamic smoothness requirements. A smooth blend is also generated between the functional area and the unperturbed fuselage.

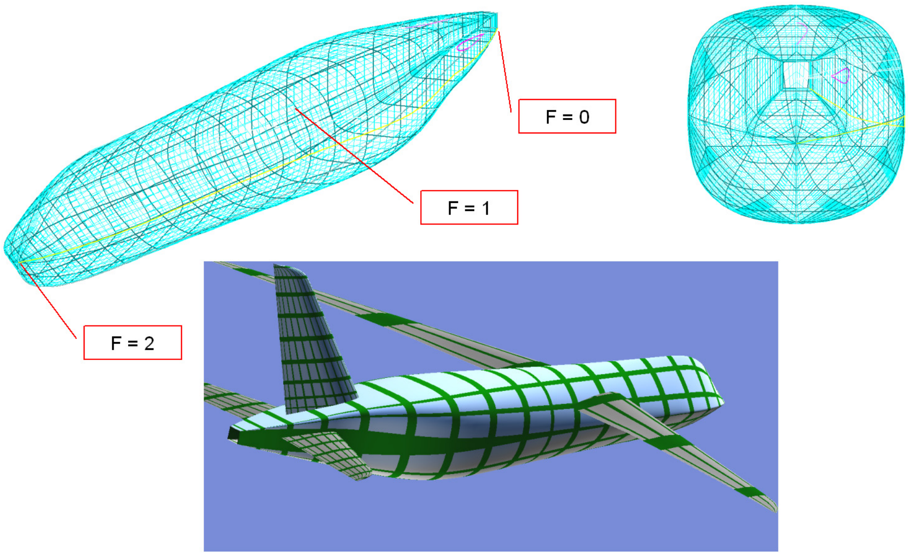

Figure 4: Example of an extreme definition of a fuselage by progressive shape transformation of a basic circular cross section. The shape factor (F) of 2 at the nose corresponds to a purely rhomboidal section. F=1 corresponds to the unperturbed circular cross section at mid length. F=0 transforms the basic section into a square.

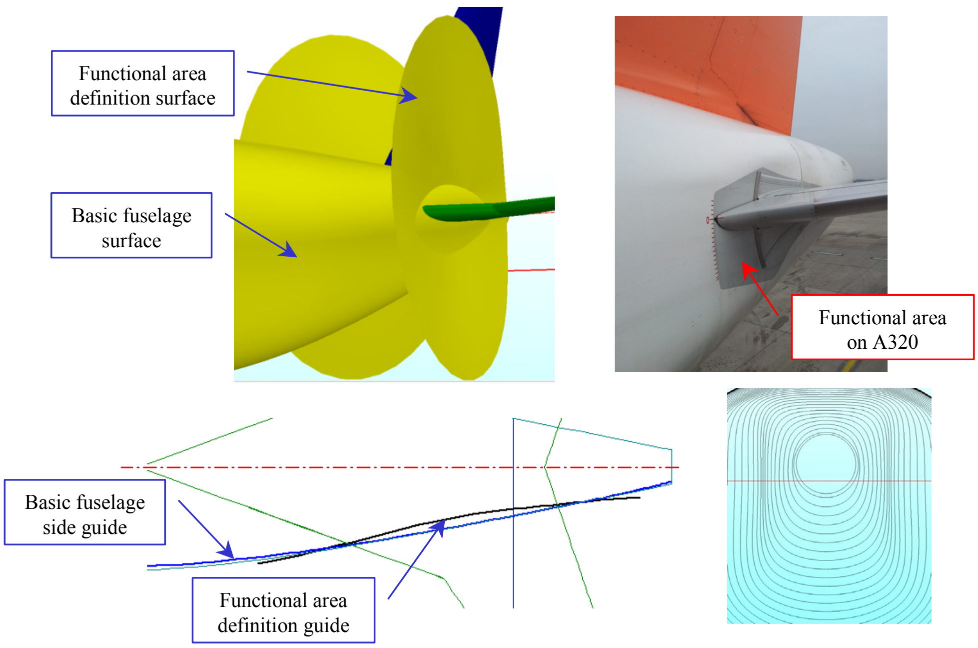

Figure 5: Geometric definition of a functional area for a trimmable horizontal tailplane

2.2 Lifting surfaces parametric geometry

The geometry of the lifting surfaces is generated by MARES by a method of shape modulation of a topological tube. This is a similar approach to that applied in the fuselage but with significant differences in the case of lifting surfaces.

The cross sections of the lifting surfaces are defined by aerofoils imported from an external catalogue. The master aerofoils are defined at particular stations and the non-dimensional shape of intermediate sections is interpolated using smooth functions with curvature continuity.

The planform of the lifting surfaces is supported on a single trapezium. Four independent parameters are required to define a trapezoidal planform, for example: span, root chord, tip chord and sweep of the 25% chord line. Alternative parameters often used are; reference surface, aspect ratio, taper ratio and sweep of the 25% chord line. Both definitions can be used in MARES.

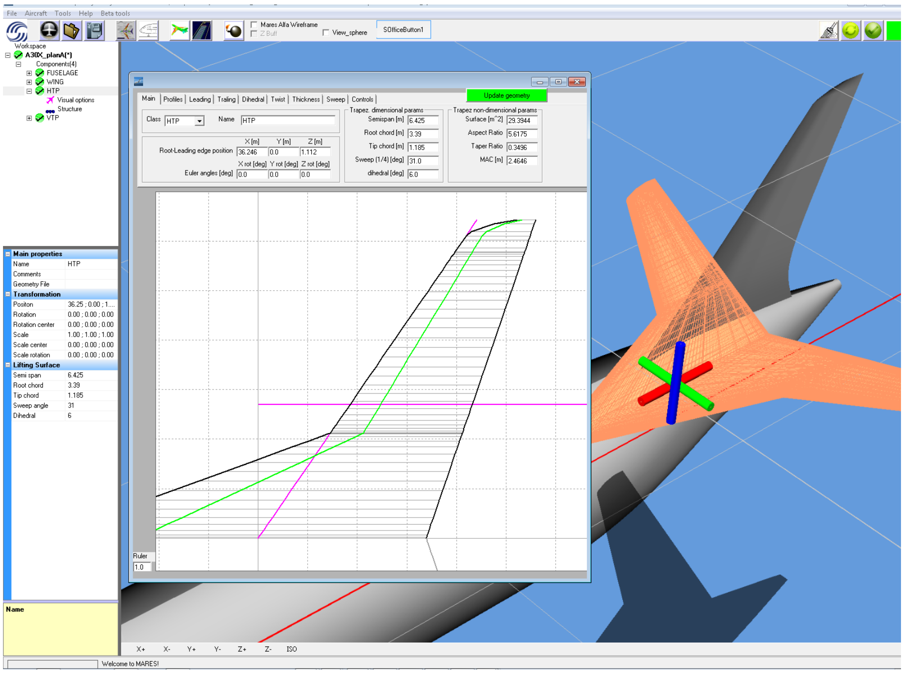

Figure 6: Planform definition of a lifting surface

A single trapezium may be sufficient to represent the planform of a typical tail surface. When a more detailed definition is required, for example to represent a Küchemann tip or, more generally, any curved or kinked leading or trailing edges, perturbation functions are applied on the basic trapezoidal planform. The perturbations are expressed in terms of the local non-dimensional trapezium chord, x/c.

Any number of master stations can be defined in the spanwise direction at the leading or trailing edge where the value of the perturbations is set. Between the master stations, the perturbation function is interpolated using a smooth curve with particular end tangent constraints. In this manner, both linear and curved perturbation curves can be defined, allowing great flexibility in the definition of the actual planform.

As the planform perturbation control points and parameters are defined in non-dimensional form, a change to any of the principal trapezoidal parameters will be automatically reflected in the final geometry. Therefore, a complete scaling of the surface, a change in taper, aspect ratio or sweep angle will be automatically reflected in the true planform. In particular cases, the actual locations of the control points or the sweep angles of the leading or trailing edges must be defined in dimensional form, which is enabled in MARES by automatically adjusting the non-dimensional parameters in a local constraint solving process to obtain the desired result.

For complete generality, the geometry generation of the lifting surfaces must be capable of representing dihedral, torsion and variable absolute aerofoil thickness. The spanwise variation of all these parameters is defined in MARES also at key stations and the intermediate interpolation follows the same philosophy as for the planform perturbations. An axis is required in order to perform the twist transformation of the intermediate aerofoil sections and this can also be defined by the user.

The previous methodology provides great generality and flexibility to represent generic lifting surfaces of high quality, an example of which, including the results of an aerodynamic analysis, can be seen in figure 7.

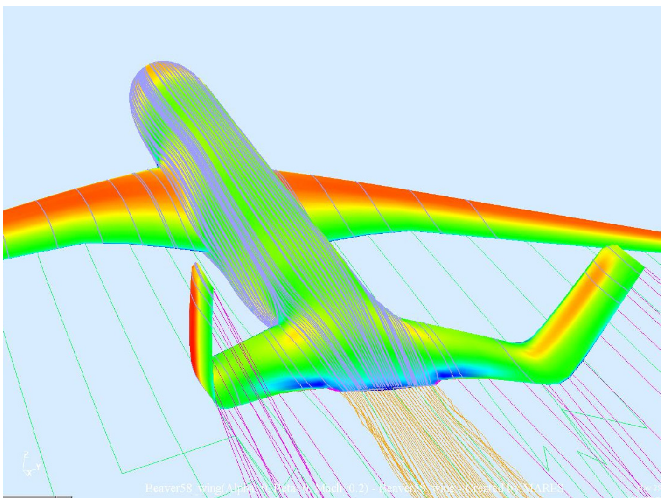

Figure 7: Example of tail surfaces generated in MARES by interpolation of aerofoils, chord adaptation, twist and dihedral transformation

2.4 Miscellaneous aircraft components

In order to perform space allocation, ground clearance analysis and various other studies based on the aircraft geometry, MARES enables the definition of various aircraft components and systems. In these cases, the focus is on space allocation and clash detection and the level of detail is reduced compared with the main elements required for the aerodynamic analysis of the rear fuselage and tails.

An important element usually located in the tailcone of conventional aircraft is the Auxiliary Power Unit (APU). The geometry of the APU and its fire compartment can impose a design constraint for the tailcone and aft fuselage shapes. A simplified parametric model is included in MARES which provides sufficient detail for space allocation analysis and external geometry definition.

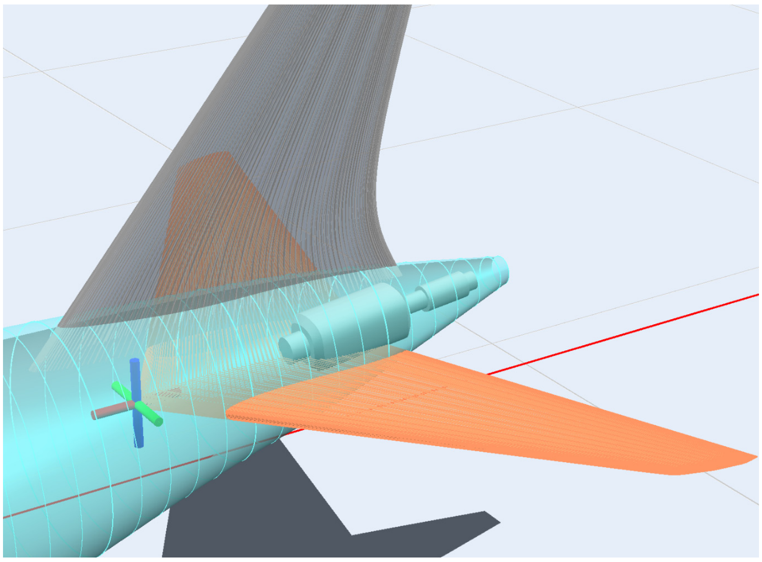

Figure 8: Parametric geometry of auxiliary power unit (APU)

A particularly important geometric constraint on the rear fuselage is the tail-strike angle. The rotation of the aircraft at take-off must be considered in the design of the rear fuselage and is influenced by the position and geometry of the landing gear. In order to perform a tail-strike angle analysis automatically, a simplified model of the landing gear tyres is provided in MARES.

2.5 Surface Quality analysis

The assessment of the surface quality and its characteristics is an important part of the design process, particularly when the intention is to perform mid to high fidelity aerodynamic analysis on the surfaces generated by MARES.

Various methods are implemented to facilitate the inspection of the surface quality. In interactive design mode, the curvature of the fuselage guide sections can be plotted which enables the identification of zones of high or discontinuous curvature.

Once the 3D surfaces have been instantiated, a zebra stripping plot is provided as “zeroth-order” surface evaluation. This is just a directional mapping of a stripped pattern and allows to detect kinks in the surface.

A more interesting surface evaluation method is the representation of the value of the angle formed by the local surface normal with a given direction. In the case of a fuselage, the most valuable information is obtained when using the longitudinal axis as reference. This method can be considered of first-order, related to the directional derivative of the surface, and enables to detect certain critical undulations in the surface and assess its general smoothness. In MARES, the resulting plots are referred to as “isogonic contours”.

Finally, a Gaussian curvature plot is available to assess local surface curvature and identify changes in convexity and regions where the surface is developable.

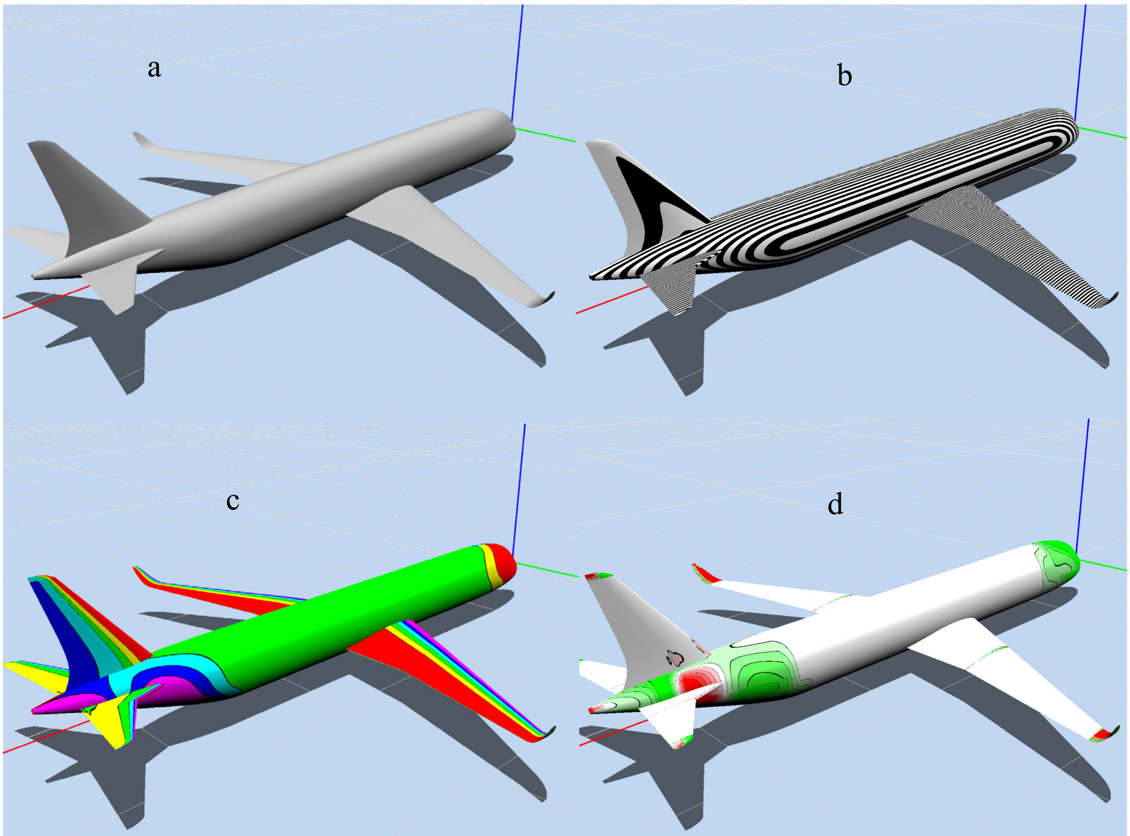

Figure 9: Typical aircraft geometry: Shaded (a), zebra stripping (b), x-isogonic contours (c), Gaussian curvature (d). All rendering directly from MARES.

2.6 Geometry import and export

Although the geometry generation in MARES uses analytical functions throughout and therefore the surfaces can be evaluated at any point with arbitrary precision, the instantiated geometric model in MARES is a tessellated mesh. This model is easily exported into any tessellated 3D file format, like VRML or STL. Conversely, external geometry models using a tessellated mesh can be imported and visualised in MARES.

The tessellated geometry generated by MARES is processed internally to generate space allocation models, aerodynamic and structural meshes. It is, however desirable to export the geometry and the structural arrangement also into a CAD model. This capability has been implemented in MARES by communicating with an external CAD package through its application programming interface (API). The CAD geometric entities used are compatible with the internal definition of the surfaces in MARES and in order to represent the internal structure, datum planes are constructed and the intersections are performed using the CAD package, all of this driven automatically by MARES.

An additional capability exists to generate NURBS surfaces in MARES for specific purposes (e.g., aerodynamic fairings) and these entities can be exported directly into a CAD format via an IGES file.

3 Aerodynamic Model Generation

The aerodynamic characteristics of tails surfaces are fundamental drivers of their size and planform. In order to provide stability to the aircraft, tail surfaces act as aerodynamic springs, generating a restoring force when their angle of attack is changed. The rate of lift force generation for variations of angle of attack is called lift slope or stability derivative. Tail surfaces also provide the means to change the attitude or sideslip angles of the aircraft in their function as control surfaces through the deflection of elevators or rudders. The rate of generation of control force or deflection of the controls is called control derivative. The stability and control derivatives depend on the planform of the tails and the actual aerodynamic force is proportional to the size of the surface and the angle of attack or control deflection.

As the aircraft wing approaches the stall angle, the horizontal tail must provide the sufficient restoring force to bring the aircraft back to level flight. Therefore, the horizontal tailplane cannot stall before the wing and, thus, the tail stall angle is also an important design consideration when selecting the planform shape, which is, together with the aerofoil shape, the main driver of the stall characteristics.

From the previous discussion it is clear that an analysis driven design process for tail surfaces requires a means to evaluate the relevant aerodynamic responses of a given tail planform and size. Given the geometric constraints imposed by -and on-the fuselage and since many of the "sizing" cases correspond to low speed flight conditions, where compressibility effects are not significant, a 3D potential panels method coupled with an integral boundary layer formulation is deemed to provide the best compromise between accuracy in the aerodynamic response and simplicity in terms of mesh generation and short analysis time.

The aerodynamic analysis method used by MARES is VSAERO, a commercial package of wide industrial application.

VSAERO requires a structured surface mesh of quadrilateral panels with, ideally, congruence of panels between blocks.

3.1 Mesh generation for the aerodynamic panels method

The surface mesh generation process starts with the calculation of all the intersections between the aerodynamic components in MARES, namely fuselage and lifting surfaces.

This operation is performed on the 3D geometry and is facilitated by the fact that all components are built with bi-parametric patches.

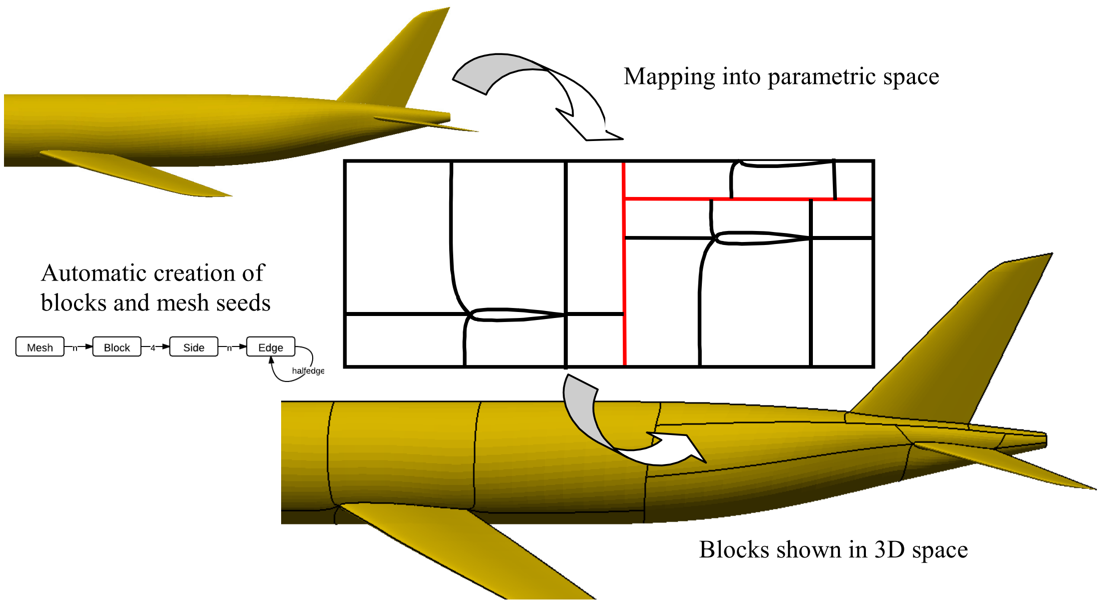

Figure 10: Structured multiblock surface mesh generation process

Once the intersections are available, a mapping is carried out between 3D coordinates and the bi-parametric space, where the intersected components are rectangles and the intersections are closed loops with a cusp point corresponding to the trailing edge of the aerofoils.

The intersection loops are identified and the intersected component is divided in blocks according to the selected mesh topology. Particular attention is paid to the geometry of the block boundaries at the leading edge of the aerofoil intersections.

With the blocks defined and their boundaries calculated, a process of "negotiation" between blocks is carried out in order to seed the block edges with the same number of panels distributed in a congruent arrangement. In order to improve the numerical solution, the panels are seeded at higher density near the leading and trailing edges of the intersected aerofoils.

Once the congruency at the block edges has been guaranteed, the structured mesh is generated by creating internal points in each block using an iso-parametric mapping. In order to improve the quality of the mesh, a number of iterative Laplacian mesh smoothing loops are applied. The effect of the smoothing process is to "relax" the mesh and reduce the number of panels with high aspect or taper ratio.

The final phase of the mesh generation process is to revert to 3D geometric space by converting the parametric coordinates (u, v) of the internal mesh nodes into true coordinates (x,y,z). At this point, a generic structured multi-block surface mesh of generally very good quality is available.

VSAERO requires an explicit definition of the wakes. Further operations are required to define the free wake and, particularly, the wake fixed on the boundary of the intersected panels. These operations are performed automatically in 3D space by MARES.

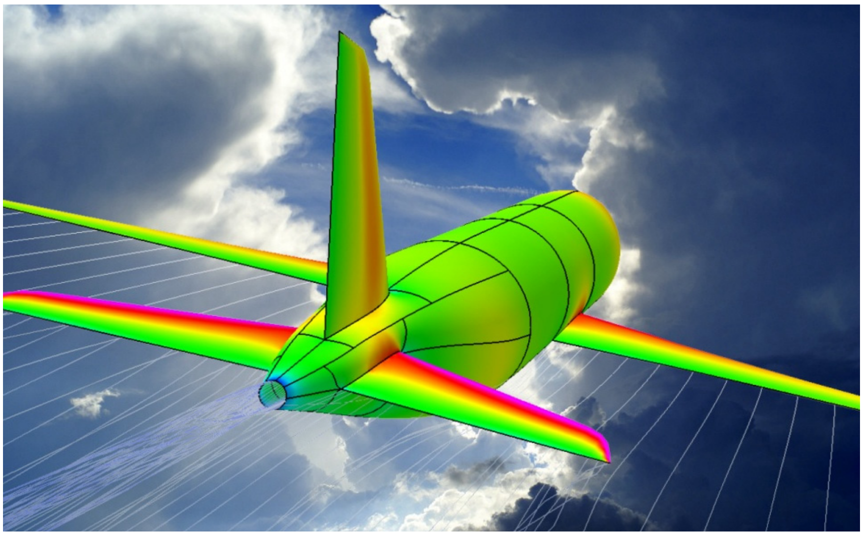

Figure 11: Results from the VSAERO analysis: Pressure coefficient plot and relaxed wake geometry

Finally, an input file containing the mesh and wakes definition, the numerical solver parameters and the flight conditions is automatically written and VSAERO launched.

The control surfaces (elevator in the horizontal tailplane and rudder in a vertical tailplane) are defined in planform view. The edges of the controls are mapped onto the 3D surface and converted into bi-parametric space as block boundaries. The mesh generation then proceeds as described before.

The results of the aerodynamic analysis in terms of stability and control derivatives are obtained directly from the output file. The stall angle is calculated using a methodology based on the gradient of pressure coefficient (Cp) distributions. When the design is iterated manually, inspection of the Cp plots enables to drive the external shapes in order to obtain the desire flow field.

3.2 Unsteady aerodynamic analysis of MARES geometries using a "meshless" Lattice-Botzmann method

Aerodynamic panels methods are somehow restricted in their applicability, although represent a useful design tool due to their speed of analysis and generality within their range of applicability (linear aerodynamic, low speed regime).

Emerging "meshless" CFD methods may become a fundamental design tool in the future. The lattice-Boltzmann method (LBM), based on statistical mechanics and, in principle even more general than the Navier-Stokes equations, is a good candidate to offer the sought flexibility for the aerodynamic analysis of complex geometries without the need for a traditional structured mesh tightly adapted to the external surfaces. An implementation of the LBM method in the commercial package XFlow automatically generates a Cartesian, locally refined mesh with dynamic adaptation to the flow using only a tessellated representation of the external geometry of the object under analysis.

The geometries generated by MARES fulfill the requirements for such LBM analysis and the facility to export the required geometric models has been implemented. The results of an unsteady LBM-CFD analysis on a MARES generated geometry are shown in figure 12.

4 Conclusions

An automatic parametric geometry generator has been developed, including a simple but powerful geometry engine, with the objective of creating high quality surface models of aircraft components from a reduced set of design parameters, with special focus on the rear fuselage and tail surfaces of conventional commercial aircraft. The topology of the geometric entities is particularly well suited for the subsequent automatic generation of a structured mesh for an aerodynamic panels method. The geometric model generated by MARES is also well adapted for the direct analysis by a LMB-CFD method and an example is provided.

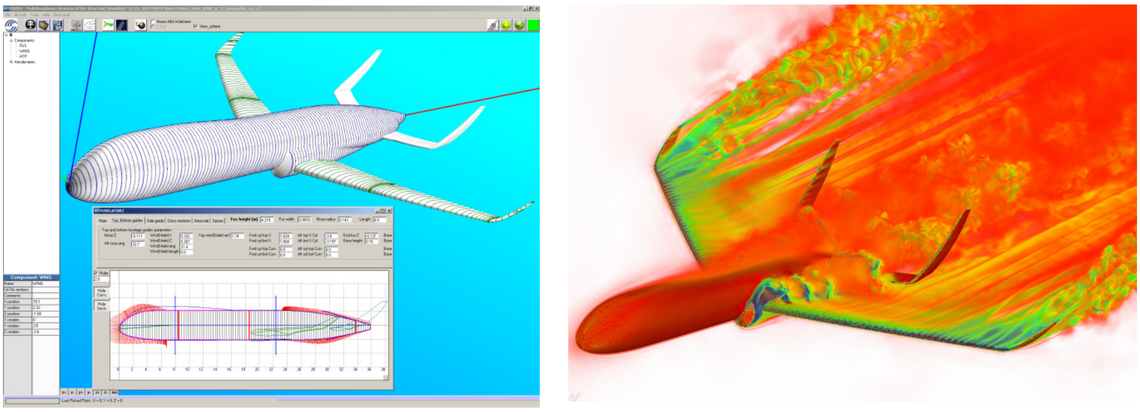

Figure 12: Unconventional geometry as generated in MARES (left), direct Lattice-Boltzmann unsteady CFD analysis with XFlow (right)

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