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# Flexible Multibody Systems with Active Vibration Control

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## Abstract

In this paper the flexible multibody dynamics formulations of complex models including elastic components made of composite materials, which may be laminated and anisotropic, are extended to include piezoelectric transducers for sensing and actuation purposes. Electromechanical coupling models of the surface-bonded piezoelectric transducers with the flexible multibody components are taken into account on the spatial models. These electromechanical effects are introduced in the flexible multibody equations of motion by the use of advanced finite plate or shell element, which is specially developed to this purpose. A comparison between classical, constant gain and constant amplitude velocity feedback, and optimal, linear quadratic regulator, control strategies is performed in order to investigate their effectiveness to suppress vibrations in piezoelastic smart structures undergoing rigid body motion.

**Keywords:** piezoelectric material, active control, flexible multibody systems, elastic coupling, mode component synthesis.

# **1** Introduction

The potential of application of active control strategies to flexible structures subjected to large rigid body motion is very high. In most cases the vibrations that take place during the operation of mechanical systems influence their functionality. These unwanted vibrations lead to excessive mechanical energy dissipation decreasing the efficiency, capacity and durability of machines and devices. Moreover, large variable stresses accompanied by vibration phenomena often lead to strain cracks of machine elements and to their break-down.

Several approaches have been developed to improve the elastodynamic response of flexible systems [1-7]. From the control point of view, the flexible multibody

systems can inherit have some of the basic control strategies used for other mechanical systems, such as worms [8] or vehicles [9] or even in the optimal design of underactuated mechanisms [10]. A great number of attempts have been undertaken to introduce smart material devices for the active vibration control technology of mechanical systems [7, 11, 12]. Most of the efforts reported to model the flexible multibody system dynamics in the framework of classic control applications neglect the inertia coupling between flexibility and rigid body motion by considering the system elastodynamics simply superimposed to the rigid body motion [13]. Nevertheless, the limitations of the kineto-elastodynamics approach has been thoroughly demonstrated in the literature and no reason exists to continue using it [14, 15]. Alternatively, the use of the finite element method together with the floating or the absolute body frame to model flexible mechanisms, leads to dynamic formulations that include the coupling of rigid body motion with the body elastodynamics. The floating reference frame method and the absolute reference frame method are two categories of modeling methods that may be used to include the coupling effects. The first method is suitable for cases in which linear elastic displacements are assumed, which has been applied in a variety of applications such as vehicle dynamic simulations [16], railway vehicles [17] or satellite solar panel deployment [18], just to mention a few. The second approach has been used when nonlinear deformations are present in the system components [14, 19, 20]. Moreover, the formulations used for the description of large motion of flexible members have been used in the framework of systems made of standard materials and composite materials [21, 22] while the work now presented is a contribution to the representation of flexible multibody systems using smart composite materials. In particular, it is presented a methodology to describe smart composite beam and plate/shell elements that follows the work proposed by [22].

Various strategies to actively control the vibrations of structures with piezoelectric layers, acting as sensors or actuators, have been applied [23]. Feedback control algorithms can be used in the active vibration control of flexible components in which the excitation of the structure cannot be directly observed and, thus, cannot be used as a feedforward control signal. In this work a comparative study between classical and optimal feedback control strategies on the active control of vibrations of structural component subjected to rigid body motions is carried in the process of demonstrating the use of the proposed methods. The presented methodology is applied to illustrative examples where the active vibration control of flexible members of a multibody system is implemented.

### 2 Multibody Systems with Smart Components

The analysis of these kind of systems imply the use of a formulation that must be consistent with the linear piezoelectric material constitutive relations. Thus, the flexible multibody methodology must account for the mechanical actuation loadings produced by the piezoelectric actuator under an input voltage, which is a distinctive feature relatively to common formulations of flexible multibody dynamics.

### 2.1 Formulation of a smart Multibody system

A rigid body is defined by the position of a body fixed reference frame,  $\xi \eta \zeta$ , and its orientation with respect to an inertia frame, XYZ. The position and orientation of the rigid body *i* is defined by a set of translation coordinates, represented by  $\mathbf{r}_i$  and rotational coordinates  $\mathbf{p}_i$ . The body coordinates are then grouped in a vector  $\mathbf{q}_{ri} = [\mathbf{r}_i^T \mathbf{p}_i^T]^T$ . The coordinate vector of the complete system, designated by  $\mathbf{q} = [\mathbf{q}_r^T \mathbf{q}_f^T]^T$ , is composed of the coordinate vector of the individual bodies  $\mathbf{q}_{ri}$  and the nodal coordinates of flexible bodies. Notice that for a multibody system composed by *n* flexible bodies,  $\mathbf{q}_f$  includes vector  $\mathbf{u}_i$  of the nodal or modal displacements for all system flexible bodies.

For a multibody system in which some of the mechanical components involve piezoelectric transducers in combined actuating/sensing mode, the system equations of motion are given by

$$\begin{bmatrix} \mathbf{M}_{rr} & \mathbf{M}_{rf} & \mathbf{\Phi}_{\mathbf{q}_{r}}^{T} \\ \mathbf{M}_{fr} & \mathbf{M}_{ff} & \mathbf{\Phi}_{\mathbf{q}_{f}}^{T} \\ \mathbf{\Phi}_{\mathbf{q}_{r}} & \mathbf{\Phi}_{\mathbf{q}_{f}} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}_{r} \\ \ddot{\mathbf{q}}_{f} \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathbf{g}_{r} \\ \mathbf{g}_{f} \\ \gamma \end{bmatrix} - \begin{bmatrix} \mathbf{s}_{r} \\ \mathbf{s}_{f} \\ \mathbf{0} \end{bmatrix} - \begin{bmatrix} \mathbf{0} \\ (\mathbf{K}_{ff} + \mathbf{K}_{p})\mathbf{q}_{f} \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{g}_{r}^{a} \\ \mathbf{g}_{f}^{a} \\ \mathbf{0} \end{bmatrix}$$
(1)
$$\mathbf{\phi}^{s} = \mathbf{K}^{s} \mathbf{q}_{f}$$
(2)

where

$$\mathbf{K}^{s} = \begin{bmatrix} \left\{ \left( \mathbf{K}_{\varphi\varphi}^{s} \right)^{-1} \mathbf{K}_{\varphi u}^{s} \right\}_{1} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \end{bmatrix}$$
(3)

$$\begin{bmatrix} \mathbf{0} & \cdots & \left\{ \left( \mathbf{K}_{\varphi\varphi}^{s} \right)^{-1} \mathbf{K}_{\varphi u}^{s} \right\}_{n} \end{bmatrix}$$
$$\mathbf{K}_{p} = \sum_{j} \mathbf{K}_{s}^{j} + \sum_{k} \mathbf{K}_{a}^{k}$$
(4)

where matrix  $\mathbf{K}_{p}$  is the global induced stiffness due to the piezoelectricity,  $\mathbf{K}_{s}^{j}$  and  $\mathbf{K}_{a}^{k}$  are the corresponding induced stiffness of the piezoelectric sensor in open circuit and the piezoelectric actuator, defined as

$$\mathbf{K}_{s}^{j} = \mathbf{K}_{f\varphi}^{s} (\mathbf{K}_{\varphi\varphi}^{s})^{-1} \mathbf{K}_{\varphi f}^{s}$$
(5)

$$\mathbf{K}_{a}^{k} = \mathbf{K}_{f\varphi}^{a} (\mathbf{K}_{\varphi\varphi}^{a})^{-1} \mathbf{K}_{\varphi f}^{a}$$
(6)

The superscripts *s* and *a* are used to denote the sensors that are in open circuit and the piezoelectric elements that are actuators, respectively.  $\mathbf{K}_{f\varphi}$  and  $\mathbf{K}_{\varphi f}$  are the piezoelectric coupling matrices,  $\mathbf{K}_{\varphi\varphi}$  is the dielectric permittivity matrix. In Equation (2)  $\varphi^s$  is the vector of electrical degrees of freedom and in Equation (1) vectors  $\mathbf{g}_r^a$  and  $\mathbf{g}_f^a$  are the external piezoelectric forces due to the effect of actuators, which are applied with respect to rigid body and flexible coordinates, respectively. For a flexible body i with piezoelectric patches the vector  $\mathbf{g}_{f_i}^a$  is obtained by

$$\mathbf{g}_{f_i}^a = -\left(\mathbf{K}_{u\varphi}^a \boldsymbol{\varphi}^a\right)_i \tag{7}$$

and vector  $\mathbf{g}_{r_i}^a$  is defined as

$$\mathbf{g}_{r_i}^a = \left[\sum_{k=1}^n \left(\mathbf{A}_i \mathbf{g}_{f_i}^a\right)_k \quad \sum_{k=1}^n \left(\tilde{\mathbf{b}}_i' \mathbf{g}_{f_i}^a\right)_k\right]^T$$
(8)

where  $(\mathbf{g}_{f_i}^a)_k$  is the local body actuation force vector associated to the node k of flexible body *i*,  $\mathbf{A}_i$  is the transformation matrix from the body referential to the inertial referential [24] and  $(\tilde{\mathbf{b}}'_i)_k$  is the position of node k in the body-fixed frame, represented by

$$\mathbf{b}_i^{\prime k} = \mathbf{x}_i^{\prime k} + \mathbf{\delta}_i^{\prime k} \tag{9}$$

In Equation (8) the vector  $\mathbf{\delta}_{i}^{\prime k}$  is used to represent the nodal deformation that is time dependent and  $\mathbf{x}_{i}^{\prime k}$  is the nodal undeformed position that is independent of time

The equations of motion for the flexible multibody system in the form used in Equation (1) require a large number of coordinates to describe complex models. Using a component mode synthesis method each flexible body may be described by the relation presented by

$$\mathbf{u} = \mathbf{x} \,\boldsymbol{\eta} \tag{10}$$

Thus, the reduced equations of motion for the flexible multibody systems are

$$\begin{bmatrix} \mathbf{M}_{rr} & \mathbf{M}_{rf} \mathbf{X} & \mathbf{\Phi}_{\mathbf{q}_{r}}^{T} \\ \mathbf{X}^{T} \mathbf{M}_{fr} & \mathbf{I} & \mathbf{X}^{T} \mathbf{\Phi}_{\mathbf{q}_{f}}^{T} \\ \mathbf{\Phi}_{\mathbf{q}_{r}} & \mathbf{\Phi}_{\mathbf{q}_{f}} \mathbf{X} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}_{r} \\ \ddot{\mathbf{w}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{g}_{r} \\ \mathbf{X}^{T} \mathbf{g}_{f} \\ \overline{\boldsymbol{\gamma}} \end{bmatrix} - \begin{bmatrix} \mathbf{s}_{r} \\ \mathbf{X}^{T} \mathbf{s}_{f} \\ \mathbf{0} \end{bmatrix} - \begin{bmatrix} \mathbf{0} \\ \mathbf{A} \mathbf{w} \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{g}_{r}^{a} \\ \mathbf{X}^{T} \mathbf{g}_{f}^{a} \\ \mathbf{0} \end{bmatrix}$$
(11)

$$\boldsymbol{\varphi}^{s} = \mathbf{K}^{s} \mathbf{X} \mathbf{w} \tag{12}$$

where

$$\mathbf{w} = \begin{bmatrix} \boldsymbol{\eta}_1 & \dots & \boldsymbol{\eta}_n \end{bmatrix}^T; \ \mathbf{X} = \begin{bmatrix} \mathbf{x}_1 & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{x}_n \end{bmatrix} \mathbf{\Lambda} = \begin{bmatrix} \boldsymbol{\Psi}_1 & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \boldsymbol{\Psi}_n \end{bmatrix}$$
(13)

In Equation (13)  $\Psi_i$  is a diagonal matrix that contains the squares of natural frequencies associated to the modes of vibration, , of flexible body i. Due to the reference conditions, the modes of vibration used here are constrained modes and, due to the assumption of linear elastic deformations the modal matrix is invariant [14].

The structural damping is added to flexible bodies, by means of proportional damping, resulting in an improvement on the performance of the numerical integrator and in no significant change in the simulation results. For this purpose the equations of motion of a mechanical system with flexible bodies exhibiting structural damping are modified to include a damping term, yielding

$$\begin{bmatrix} \mathbf{M}_{rr} & \mathbf{M}_{rf}\mathbf{X} & \mathbf{\Phi}_{\mathbf{q}_{r}}^{T} \\ \mathbf{X}^{T}\mathbf{M}_{fr} & \mathbf{I} & \mathbf{X}^{T}\mathbf{\Phi}_{\mathbf{q}_{f}}^{T} \\ \mathbf{\Phi}_{\mathbf{q}_{r}} & \mathbf{\Phi}_{\mathbf{q}_{f}}\mathbf{X} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}_{r} \\ \ddot{\mathbf{w}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{g}_{r} \\ \mathbf{X}^{T}\mathbf{g}_{f} \\ \overline{\boldsymbol{\gamma}} \end{bmatrix} - \begin{bmatrix} \mathbf{s}_{r} \\ \mathbf{X}^{T}\mathbf{s}_{f} \\ \mathbf{0} \end{bmatrix} - \begin{bmatrix} \mathbf{0} \\ A\mathbf{w} \\ \mathbf{0} \end{bmatrix} - \begin{bmatrix} \mathbf{0} \\ (\alpha \mathbf{\Lambda} + \beta \mathbf{I})\dot{\mathbf{w}} \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{g}_{r}^{a} \\ \mathbf{X}^{T}\mathbf{g}_{f}^{a} \\ \mathbf{0} \end{bmatrix}$$
(14)

Note that the only difference between Equations (11) and (14) resides on the terms of the right-hand-side.

### 2.2 Active control of vibrations

#### 2.2.1 Classical control

The aim of this section is to design a controller in which the control voltage is given by

$$\boldsymbol{\varphi}^{a} = \begin{bmatrix} -\mathbf{G}_{d} & -\mathbf{G}_{v} \end{bmatrix} \begin{bmatrix} \boldsymbol{\varphi}^{s} \\ \dot{\boldsymbol{\varphi}}^{s} \end{bmatrix}$$
(15)

where  $\mathbf{G}_d$  and  $\mathbf{G}_v$  are feedback gain matrices defined according the selected control law. The first order derivative of Equation (12) can be used to define vector  $\dot{\boldsymbol{\phi}}^s$  as

$$\dot{\boldsymbol{\varphi}}^{s} = \mathbf{K}^{s} \mathbf{X} \, \dot{\mathbf{w}} \tag{16}$$

Using Equations (15), (12) and (16) and the relation presented in Equation (7), it is possible to define the actuation force vector for a flexible multibody system by

$$\mathbf{g}_{f}^{a} = \begin{bmatrix} -\mathbf{G}_{w} & -\mathbf{G}_{\dot{w}} \end{bmatrix} \begin{bmatrix} \mathbf{w} \\ \dot{\mathbf{w}} \end{bmatrix}$$
(17)

where

$$\mathbf{G}_{w} = \begin{bmatrix} \left\{ \mathbf{K}_{u\varphi}^{a} \, \mathbf{G}_{d} \left( \mathbf{K}_{\varphi\varphi}^{s} \right)^{-1} \, \mathbf{K}_{\varphiu}^{s} \, \mathbf{X} \right\}_{1} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \left\{ \mathbf{K}_{u\varphi}^{a} \, \mathbf{G}_{d} \left( \mathbf{K}_{\varphi\varphi}^{s} \right)^{-1} \, \mathbf{K}_{\varphiu}^{s} \, \mathbf{X} \right\}_{n} \end{bmatrix}$$
(18)  
$$\mathbf{G}_{w} = \begin{bmatrix} \left\{ \mathbf{K}_{u\varphi}^{a} \, \mathbf{G}_{v} \left( \mathbf{K}_{\varphi\varphi\varphi}^{s} \right)^{-1} \, \mathbf{K}_{\varphiu}^{s} \, \mathbf{X} \right\}_{1} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \left\{ \mathbf{K}_{u\varphi}^{a} \, \mathbf{G}_{v} \left( \mathbf{K}_{\varphi\varphi\varphi}^{s} \right)^{-1} \, \mathbf{K}_{\varphiu}^{s} \, \mathbf{X} \right\}_{n} \end{bmatrix}$$
(19)

If a velocity feedback scheme is utilized, the sensor output is differentiated, amplified and then used to feed back the actuator. In this control strategy, the signal used is representative of the strain rate of the structural component and, therefore, is named velocity feedback control. For this case, two control algorithms are analyzed [35]: the constant amplitude velocity feedback (CAVF); and the constant gain velocity feedback (CGVF). In the first control algorithm, the individual gain of the jth piezoelectric actuator is defined according to the polarity of the jth sensor voltage and to denote this fact the function sign() is used. The amplitude of the feedback control voltage is constant, non-linear or discontinuous. The gain matrix for a flexible body i is defined by

$$\left(\mathbf{G}_{d}\right)_{i} = \mathbf{0}; \left(\mathbf{G}_{v}\right)_{i} = \operatorname{diag}(A_{1}, \dots, A_{j}, \dots, A_{n})$$

$$(20)$$

where diag() denotes a diagonal matrix with the individual amplitudes  $A_j$  of the *j*th actuating control voltage. For the flexible body *i*, the control voltage is given by

$$\mathbf{\phi}_{i}^{a} = -\left(\mathbf{G}_{v} sign(\dot{\mathbf{\phi}}^{s})\right)_{i} \tag{21}$$

Alternatively, for a constant gain velocity feedback control algorithm the gain matrix associated with the flexible body i is defined by

$$\left(\mathbf{G}_{d}\right)_{i} = \mathbf{0}; \ \left(\mathbf{G}_{v}\right)_{i} = \operatorname{diag}(G_{1}, \dots, G_{j}, \dots, G_{n})$$

$$(22)$$

where  $A_j$  is the individual gain of the *j*th actuator associated with the deformation sensor and  $G_j$  is the individual gain of the *j*th actuator associated with the strain rate deformation of sensor. Nevertheless, the control voltage is still given by Equation (15).

#### 2.2.2 Optimal control

In optimal control the feedback control system is designed to minimize a cost function or performance index, which can be written in the form

$$J = \int_{0}^{t_{f}} \left[ \mathbf{z}^{T}(t) \mathbf{Q}_{z} \mathbf{z}(t) + \left( \mathbf{\phi}^{a}(t) \right)^{T} \mathbf{R} \mathbf{\phi}^{a}(t) \right] + \mathbf{z}^{T}(t_{f}) \mathbf{S}_{z} \mathbf{z}(t_{f})$$
(23)

where  $\mathbf{Q}_z$ , **R** and  $\mathbf{S}_z$  are the state variable, control inputs and terminal state condition positive semi-definite weighting matrices, respectively. In Equation (23) vector  $\mathbf{z}(t)$  contain the state variables defined by

$$\mathbf{z}(t) = \begin{bmatrix} \mathbf{\eta}(t) & \dot{\mathbf{\eta}}(t) \end{bmatrix}^T$$
(24)

The cost function presented in Equation (24) is minimized, for the linear timeinvariant system based on the dynamic equations of structural components, by the state feedback gain matrix given by

$$\boldsymbol{\varphi}^{a}(t) = -\mathbf{K}_{g}^{*}(t)\mathbf{z}(t) \tag{25}$$

The optimal time-varying feedback gain is given by  $\mathbf{K}_{g}^{*} = \mathbf{R}^{-1}\mathbf{B}_{\varphi}\mathbf{P}(t)$ , in which  $\mathbf{P}(t)$  is the solution of the Riccati equation, defined as

$$\mathbf{P}(t) = -\mathbf{P}(t)\mathbf{A} - \mathbf{A}^{T}\mathbf{P}(t) - \mathbf{Q}_{z} + \mathbf{P}(t)\mathbf{B}_{\varphi}\mathbf{R}^{-1}\mathbf{B}_{\varphi}\mathbf{P}(t)$$
(26)

This control strategy is named linear quadratic regulator (LQR) and requires the knowledge of all optimal gain values in the complete analysis interval. Alternatively, the use of a steady-state LQR controller considerably simplifies the controller design and the analogical and digital implementation [23]. The steady-state feedback gain matrix is given by

$$\mathbf{K}_{g} = \mathbf{R}^{-1} \mathbf{B}_{\varphi} \, \mathbf{P}^{\infty} = \begin{bmatrix} \mathbf{K}_{g}^{d} & \mathbf{K}_{g}^{v} \end{bmatrix}$$
(27)

where  $\mathbf{P}^{\infty}$  is the steady-state solution, of the Riccati equation. Using Equations (24), (25) and (27) and the relation shown in Equation (7) it is possible to define the actuation force vector for a flexible multibody system by Equation (17), in which  $\mathbf{G}_{w}$  and  $\mathbf{G}_{w}$  are

$$\mathbf{G}_{w} = \begin{bmatrix} \left\{ \mathbf{K}_{u\varphi}^{a} \mathbf{K}_{g}^{d} \right\}_{1} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \left\{ \mathbf{K}_{u\varphi}^{a} \mathbf{K}_{g}^{d} \right\}_{n} \end{bmatrix}$$
(28)  
$$\mathbf{G}_{w} = \begin{bmatrix} \left\{ \mathbf{K}_{u\varphi}^{a} \mathbf{K}_{g}^{v} \right\}_{1} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \left\{ \mathbf{K}_{u\varphi}^{a} \mathbf{K}_{g}^{v} \right\}_{n} \end{bmatrix}$$
(29)

In this regulator gain design, it is assumed that all of the states are available for feedback. In practice, only some outputs of each flexible body can be known and measured. To estimate the state variables from a model of the system and a limited number of observations of the outputs, a suitable state estimator must be designed. The state estimator must also perform well in the presence of sensor noise and other extraneous disturbances. Nevertheless, in this work the control philosophy is only based on the design of a linear quadratic regulator

## **3** Application Example

The first application used to perform classical and optimal active vibration controls is proposed by Vasques and Rodrigues [23]. It consists in a cantilever aluminum beam with an asymmetric collocated piezoelectric sensor/actuator pair mounted on the beam surface, as shown in Figure 1. The cantilever beam is 400 mm long, 2 mm thick and 15 mm wide; the two piezoelectric patches are 30 mm long, 1 mm thick and 15 mm wide. The piezoelectric patches are mounted at a distance of 5 mm from the clamped edge.



Figure 1: Cantilever smart piezoelectric beam.

The mechanical and electrical material properties of the beam are presented in Table 1. In the numerical simulation a reduced modal model with the first six modes is considered. The beam finite element model is obtained using the smart plate/shell finite element with six mechanical degrees of freedom per node and one electrical degree of freedom per element [24]. The third natural frequency accounts for the inplane bending and, therefore, only five out-of-plane flexural modes are considered. The damping coefficients presented in Equation (30) are evaluated assuming that the damping ratios of the first and fifth modes are 1.71% and 0.41%, respectively. Furthermore, the beam is discretized into 320 elements of equal dimension and a shear correction factor of 5/6 is used. The beam structure is excited during the first 0.02 s of the simulation by a mechanical force applied at the beam free edge to obtain a tip displacement of 1.5 mm. After this initial stage the mechanical force is set to zero and the beam is left free to vibrate.

Alum.			PXE-5		
Е	70 GPa	$c_{11}^{E}$	131.1 GPa	<i>d</i> <sub>31</sub>	$-215 \times 10^{-12} \text{ mV}^{-1}$
ν	0.3	$c_{12}^{E}$	7.984 GPa	<i>d</i> <sub>33</sub>	$500 \times 10^{-12} \text{ mV-1}$
ρ	2710Kgm- 3	$c_{13}^{E}$	8.439 GPa	<i>d</i> <sub>15</sub>	$515 \times 10^{-12} \text{ mV-1}$
		$c_{33}^{E}$	12.31 GPa	$\epsilon_{11}^T/\epsilon_0$	1800
		$c_{44}^E$	2.564 GPa	$\epsilon_{33}^T/\epsilon_0$	2100
		$c^E_{66}$	2.564 GPa	density	7800 Kgm-1

Table 1. Material properties of aluminum and PXE-5

The tip displacement history and the control voltage for the CGVF and CAVF control systems are presented in Figure 2. In order to compare these results with those presented by Vasques and Rodrigues [23], a control gain G=0.4 for the CGVF and a constant amplitude A=250V are selected. For the CAVF the control voltage is turned on at t=0.02s and turned off at t=0.2s. Furthermore, as performed by Vasques and Rodrigues [23], in the CAVF only the velocity polarity of the first-mode is considered in the feedback loop.



Figure 2. Tip displacement and control voltage for the cantilever beam with openloop conditions and closed-loop: (a,b) CGVF; (c,d) CAVF.

In Figure 2 it is possible to conclude that CGVF control strategy allows attenuating the free edge tip displacement with an admissible control voltage. The results presented in Figure 2 compare well with those presented in reference [23].

For the CGVF control strategy, some differences are detected; in particular, the control voltage and the tip displacement predicted here are a little smaller than those reported by the authors and, also, the frequency associated with the open-loop and the CGVF simulation results is not always the same as on the simulation results in [35].

The closed-loop 5% settling time for the CGVF and CAVF control strategies is equal to 0.36s and 0.45s, respectively, while those reported by Vasques and Rodrigues are of 0.5s and 0.58s. However, if the results are compared during 10% settling time, the CAVF control shows a faster attenuation capacity (0.24s) than the CGVF (0.28s).

In Figure 3 the simulation results obtained with the use of the LQR controller are presented and compared with those obtained with the CGVF control strategy. The LQR results are obtained with the state weighting matrices R=30 and . These matrices were defined in order to use the control voltages within the desired range. According to the limit electric field strength of the piezoelectric patches (300Vmm-1), the control voltage must be lower than 300V [23]. The use of matrix leads to control voltages that are out of the desired range.



Figure 3. Tip displacement and control voltage for the cantilever beam with closed-loop: CGVF and LQR.

From Figure 3 is possible to see that the tip displacement time history for both control strategies is rather similar. However, the tip displacement time history obtained with CGVF shows some delay in time when compared with the LQR results. This behaviour can be justified by the difference in the control voltage variation showed on both control strategies, as presented in Figure 3 (b). Because the LQR controller strategy uses a control voltage that is higher than the one used by CGVF, the decrease on the tip displacement is somehow delayed. Thus, the 5% settling time of the LQR control strategy is of 0.37s. These results compare well with those reported by Vasques and Rodrigues [23] in which the 5% settling time is of 0.34s. The LQR when compared with the velocity feedback results requires a higher control voltage

### 4 Conclusion

In this paper a general methodology for active vibration control of high-speed flexible linkage mechanisms with piezoelectric actuators and sensors has been presented. A comparison of the classical and optimal control strategies, concerning the active vibration control of smart piezoelectric beams/plates was presented. The control models assumed that one of the piezoelectric layers acts as a distributed sensor and the other one as a distributed actuator, and the sensor signal is used as a feedback reference in a closed-loop control system.

A first case study concerning the active vibration control of a cantilever beam with an asymmetric pair of collocated piezoelectric patches on the surface is analyzed. It was shown that for an initial displacement field the CGVF control strategy allows attenuating the free tip displacement with an admissible control voltage. The results presented here compare well with those presented in the literature. The differences detected are the control voltage and the tip displacement predicted, which are slightly smaller in this study than those reported in [22] and, the frequency associated with the open-loop and the CGVF simulation results, which are not always the same as in reference [23]. The CAVF control strategy with the first mode weighted showed to be a good solution, with the 10% settling time smaller than the CGVF control. The LQR regulator, when compared with the velocity feedback results, requires a higher control voltage, while providing similar results

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