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Single Electron Spin Control in Semiconductor Nanowires

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Abstract

We study the level crossing in the energy spectrum due to Rashba-Dresselhaus spin-orbit coupling in nanowires modulated by longitudinal potential. By implementing both an analytical methodology and a numerical technique based on the finite element method (FEM), we show that the level crossing point can be manipulated with the application of spin-orbit coupling in parabolic nanowires. In particular, the level crossing point can be found at larger values of k in GaAs nanowires compared to those of InAs nanowires due to large values of the Rashba-Dresselhaus spin-orbit coupling in the latter case.

Keywords: semiconductors, quantum wires, spin-orbit coupling, spintronics, perturbation theory, finite element method.

1 Introduction

Electron spin control in low dimensional semiconductor nanostructures such as quantum dots, quantum wells and quantum wires can be defined in the plane of 2 Dimensional Electron Gas (2DEG). Such control is important for spintronic logic devices, optoelectronics, quantum computing and quantum information theory [1, 2, 3, 4, 5, 6, 7]. Low dimensional semiconductors can be experimentally grown by several existing techniques such as lithographic, molecular beam epitaxy (MBE), metal organic chemical vapor deposition (MOCVD) and others. Single electron spins in these nanostructures can be manipulated by several parameters such as the gate controlled electric fields in the lateral direction and externally applied magnetic fields. The Rashba and Dresselhaus spin-orbit couplings provide another efficient way to control the single electron spins in these nanostructures [1, 3, 8]. The Rashba spin-orbit coupling arises due to structural inversion asymmetry in the crystal lattice along the growth direction [9]. The Dresselhaus spin-orbit coupling arises due to bulk inversion asymmetry in the system [10]. Intersubband-induced spin-orbit coupling interaction in quantum wells with two subbands has been introduced in Refs. [11, 12, 13]. Based on the previous studies, it is known that the induced intersubband spin-orbit coupling in quantum wells is non-zero even in symmetric quantum wells and gives rise to a non-zero spin-Hall conductivity.

In this paper, we develop a theoretical model to find the energy spectrum of the parabolic nanowire formed in the plane of 2DEG modulated by longitudinal potential. By utilizing both analytical expressions based on perturbation theory and numerical methods based on the finite element method, we show that the level crossing point can be manipulated with the application of Rashba-Dresselhaus spin-orbit coupling.

2 Mathematical Model

Our main goal is to find the crossing between the spin states due to the Rashba-Dresselhaus spin-orbit coupling in a III-V semiconductor parabolic nanowire formed in the plane of 2DEG. The schematic diagram of the geometry of the experimental device that we have in mind is similar to Ref. [6]. We consider a 1D parabolic nanowire formed by strip gates (see Fig. 1 of Ref. [6]) in the 2D plane where the truncated Fourier cosines along x-direction provide the longitudinal modulation (for example, see Eq. 2). Therefore, the total Hamiltonian of an infinite quasi-1D parabolic nanowire with uniform Rashba-Dresselhaus spin-orbit coupling in presence of longitudinal modulation potential can be written as [6]:

$$H_{xy} = \frac{\mathbf{p^2}}{2m} + \frac{1}{2}m\omega_0^2 y^2 + \frac{\alpha}{\hbar}\left(p_y\sigma_x - p_x\sigma_y\right) + \frac{\beta}{\hbar}\left(p_x\sigma_x - p_y\sigma_y\right) + \varepsilon_\nu(k), \tag{1}$$

where $\mathbf{p^2} = p_x^2 + p_y^2$ is the momentum operator, m is the effective mass, $\alpha = \gamma_R eE$ and $\beta = 0.78\gamma_D\left(2meE/\hbar^2\right)^{2/3}$ are the strengths of the Rashba and Dresselhaus spin-orbit couplings respectively, γ_R is the Rashba coefficient, γ_D is the Dresselhaus coefficient, and $\omega_0 = \hbar/m\ell_0^2$ is the confinement frequency with ℓ_0 being the oscillator strength. Also, $\sigma_i = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli spin matrices. The longitudinal modulation potential $\varepsilon_\nu(k)$ is given by

$$\varepsilon_\nu(k) = \frac{1}{2}a_0 + \sum_{j=1}^{m}\left[a_j\cos(jk/L) + a_m\cos(mk/L)\right]. \tag{2}$$

Here we choose $L = 1.2418\, nm$ and the numerical values of the coefficients a_j (meV) are 16.2873, -8.2888, 0.1492, -0.0042, 0.0002, -0.0004, -0.0002, 0.0005 for ($j = 0, 1, 2\cdots 7$). The momentum along x-direction is a good quantum number i.e., $[p_x, H_{xy}] = 0$ and we consider $\hbar k$ as the eigenvalues of the momentum operator p_x.

To find the energy spectrum of the Hamiltonian (1), it is convenient to rotate the Hamiltonian $\bar{H}_{xy} = e^{-\frac{i\pi\sigma_x}{4}} e^{\frac{-i\theta}{2}} H_{xy} e^{\frac{i\theta}{2}} e^{\frac{i\pi\sigma_x}{4}}$. The new Hamiltonian $\bar{H}_{xy}$ can be written as

$$\bar{H}_{xy} = \frac{\hbar^2 k^2}{2m} + \left(n+\frac{1}{2}\right)\hbar\omega_0 - k\gamma_{\alpha\beta}\sigma_z + \varepsilon_\nu(k) + \frac{i\gamma_{\alpha\beta}}{\ell_0\sqrt{2}}\left(a^\dagger - a\right)\left(\sigma_x \cos 2\theta - \sigma_z \sin 2\theta\right), \tag{3}$$

where $\gamma_{\alpha\beta} = \sqrt{\alpha^2+\beta^2}$, $\cos\theta = \frac{\alpha}{\sqrt{\alpha^2+\beta^2}}$, $\sin\theta = \frac{\alpha}{\sqrt{\alpha^2+\beta^2}}$. The $\sin 2\theta$ term from Eq. 3 can be removed by rotating the Hamiltonian $\tilde{H}_{xy} = e^{-\frac{i\gamma_s \hat{y}\sigma_z}{\hbar\omega_0\ell_0^2}}\bar{H}_{xy}e^{\frac{i\theta}{2}}e^{\frac{i\pi\sigma_x}{4}}$. The new Hamiltonian $\tilde{H}_{xy}$ can be written as

$$\begin{aligned}\tilde{H}_{xy} &= \frac{\hbar^2 k^2}{2m} + \left(n+\frac{1}{2}\right)\hbar\omega_0 + \frac{\gamma_s^2}{2\hbar\omega_0\ell_0^2} - k\gamma_{\alpha\beta}\sigma_z + \varepsilon_\nu(k) \\ &\quad + \frac{i\gamma_c}{\ell_0\sqrt{8}}\left[\left(a^\dagger - a - \frac{\gamma_s^2}{(\hbar\omega_0\ell_0)^2}\left(2a^\dagger + na^\dagger + a - na\right)\right)\sigma_x + \frac{\sqrt{2}\gamma_s}{\hbar\omega_0\ell_0}\sigma_y\right], \end{aligned} \tag{4}$$

where $\gamma_s = \gamma_{\alpha\beta}\sin 2\theta$, $\gamma_c = \gamma_{\alpha\beta}\cos 2\theta$ and $\hat{y} = \ell_0/\sqrt{2}\left(a + a^\dagger\right)$. It can be seen that the Hamiltonian (4) acts as a shifted parabola. Furthermore, the Hamiltonian (4) can be written as $\tilde{H}_{xy} = H_0 + H_1$ where H_0 is the diagonal part and H_1 is the non-diagonal part. Following Ref. [6], we have

$$H_0 = \frac{\hbar^2 k^2}{2m} + \left(n+\frac{1}{2}\right)\hbar\omega_0 + \frac{\gamma_s^2}{2\hbar\omega_0\ell_0^2} - k\gamma_{\alpha\beta}\sigma_z + \varepsilon_\nu(k), \tag{5}$$

$$H_1 = \frac{i\gamma_c}{\ell_0\sqrt{8}}\left[\left(a^\dagger - a - \frac{\gamma_s^2}{(\hbar\omega_0\ell_0)^2}\hat{\gamma}_a\right)\sigma_+ + \left(a^\dagger - a - \frac{\gamma_s^2}{(\hbar\omega_0\ell_0)^2}\hat{\gamma}_a\right)\sigma_-\right], \tag{6}$$

where $\hat{\gamma}_a = 2a^\dagger + na^\dagger + a - na$ and $\sigma_\pm = \sigma_x \pm \sigma_y$. For a situation where $H_0 >> H_1$, we use non-diagonal Hamiltonian H_1 as a perturbation. Based on the second order perturbation theory, the energy spectrum of the nanowire can be written as

$$\begin{aligned}\varepsilon_{k,n,+1/2} &= \frac{\hbar^2 k^2}{2m} + \left(n+\frac{1}{2}\right)\hbar\omega_0 + \frac{\gamma_s^2}{2\hbar\omega_0\ell_0^2} - k\gamma_{\alpha\beta} + \varepsilon_\nu(k) \\ &\quad - \frac{\gamma_c^2(n+1)}{8\ell_0^2}\left[\frac{\left[1 + \frac{\gamma_s^2}{(\hbar\omega_0\ell_0)^2}(1-n)\right]\left[1 - \frac{\gamma_s^2}{(\hbar\omega_0\ell_0)^2}(n+3)\right]}{\hbar\omega_0 + 2k\gamma_{\alpha\beta}}\right] \\ &\quad + \frac{\gamma_c^2 n}{8\ell_0^2}\left[\frac{\left[1 - \frac{\gamma_s^2}{(\hbar\omega_0\ell_0)^2}(2-n)\right]\left[1 - \frac{\gamma_s^2 n}{(\hbar\omega_0\ell_0)^2}\right]}{\hbar\omega_0 - 2k\gamma_{\alpha\beta}}\right], \end{aligned} \tag{7}$$

$$\begin{aligned}\varepsilon_{k,n,-1/2} &= \frac{\hbar^2 k^2}{2m} + \left(n+\frac{1}{2}\right)\hbar\omega_0 + \frac{\gamma_s^2}{2\hbar\omega_0\ell_0^2} + k\gamma_{\alpha\beta} + \varepsilon_\nu(k) \\ &\quad - \frac{\gamma_c^2(n+1)}{8\ell_0^2}\left[\frac{\left[1 + \frac{\gamma_s^2}{(\hbar\omega_0\ell_0)^2}(1-n)\right]\left[1 - \frac{\gamma_s^2}{(\hbar\omega_0\ell_0)^2}(n+3)\right]}{\hbar\omega_0 - 2k\gamma_{\alpha\beta}}\right] \\ &\quad + \frac{\gamma_c^2 n}{8\ell_0^2}\left[\frac{\left[1 - \frac{\gamma_s^2}{(\hbar\omega_0\ell_0)^2}(2-n)\right]\left[1 - \frac{\gamma_s^2 n}{(\hbar\omega_0\ell_0)^2}\right]}{\hbar\omega_0 + 2k\gamma_{\alpha\beta}}\right]. \end{aligned} \tag{8}$$

3 Results and Discussions

By utilizing both analytical and numerical techniques, in Fig. 1(a), we study the dispersion relation i.e., energy vs k for several available states in a GaAs nanowire. Here we find that the level crossing takes place approximately at $k \approx 0.4/nm$. This level crossing point is also an accidental degeneracy point that appear in the energy spectrum in Eqs. (7) and (8). It means that this level crossing point can also be theoretically investigated by using the condition $k = \hbar\omega_0/2\gamma_{\alpha\beta}$. In Fig. 1(b), we plot the wave function of electron spin states in the nanowire far away from the level crossing point i.e. at $k = 0.55/nm$. It can be seen that the first excited state wave function (red plot in Fig. 2) corresponds to the state $|k, 1, +1/2>$ which is a clear indication of the level crossing between the spin states $|k, 0, -1/2>$ and $|k, 1, +1/2>$.

In Fig. 2, we plotted energy vs k for several values of the electric fields. It can be seen that the level crossing point can be manipulated to either smaller or larger values of k with the application of Rashba-Dresselhaus spin-orbit coupling.

In Fig. 3, we study the crossing of the energy levels for InAs parabolic nanowires. In this case, the Rashba spin-orbit coupling is much stronger than the Dresselhaus spin-orbit coupling and we find the crossing point at smaller values of k compared to the corresponding values for GaAs material.

4 Conclusions

In conclusion, we have shown that the electron spin states in nanowire modulated by longitudianl potential can be manipulated with the application of Rashba-Dresselhaus spin-orbit coupling. In particular, we have shown that the level crossing point can be moved to either smaller or larger values of k with the application of electric fields that determine the strengths of the Rashba and Dresselhaus spin-orbit couplings.

Acknowledgments

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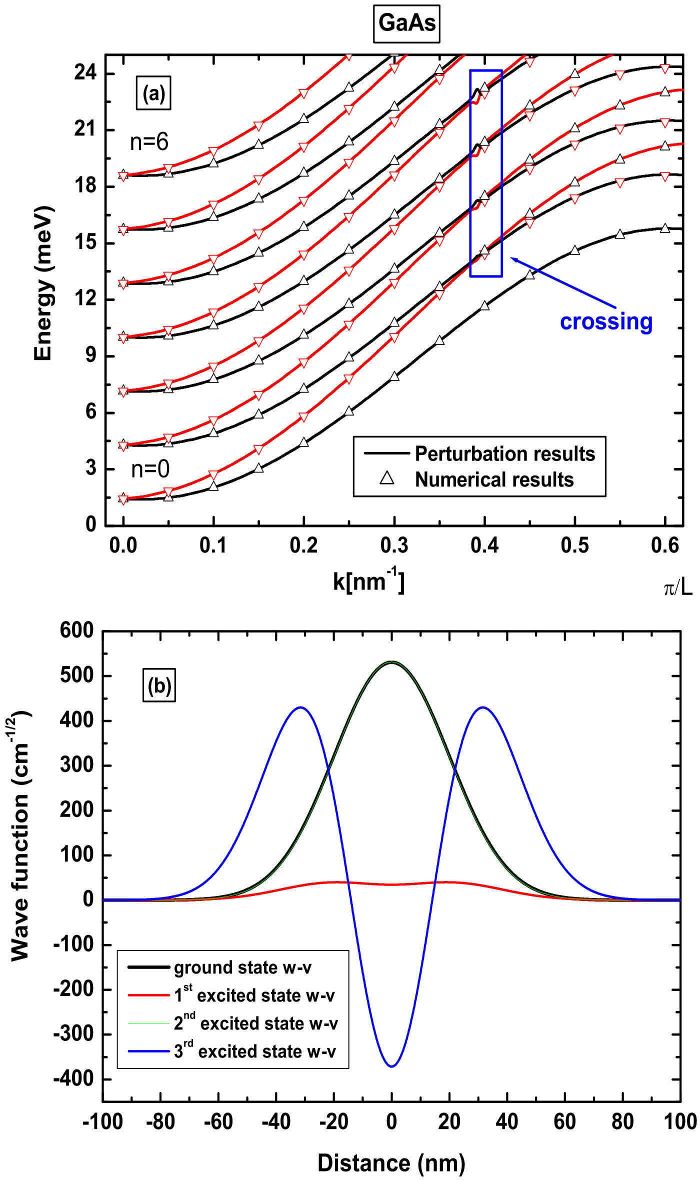

Figure 1: (Color online) (a) Dispersion relation: energy ($\varepsilon - \hbar^2 k^2/2m$) vs k. It can be seen that the energy curve corresponding to the state $|k, n, -1/2>$ crosses the energy curve corresponding to the state $|k, n+1, +1/2>$ approximately at $k = 0.4/nm$. (b) Wave function vs distance at $k = 0.55/nm$. The ground state wave function is associated to the state $|k, 0, +1/2>$ (black), the first excited state wave function is associated to the state $|k, 1, +1/2>$ (red), and the second excited state wave function is associated to the state $|k, 0, -1/2>$ (green). Here we chose $E = 3.7 \times 10^5\ V/cm$, $\ell_0 = 20\ nm$ and $\theta = \pi/6$.

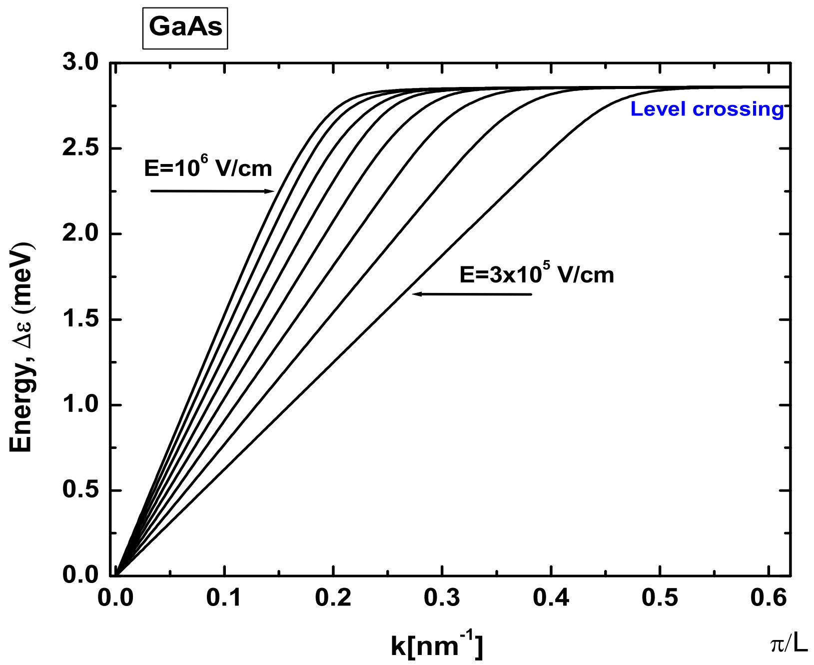

Figure 2: (Color online) Contributions of Rashba-Dresselhaus spin-orbit coupling on electron spins in a nanowire: energy ($\triangle\varepsilon = \varepsilon_{0,-1/2} - \varepsilon_{0,+1/2}$) vs k. The level crossing takes place with the accessible values of k. Here we chose $E = (3, 4 \cdots 10) \times 10^5\ V/cm$, $\gamma_R = 0.044nm^2$, $\gamma_D = 0.026eVnm^3$, $\ell_0 = 20\ nm$ and $\theta = \pi/6$.

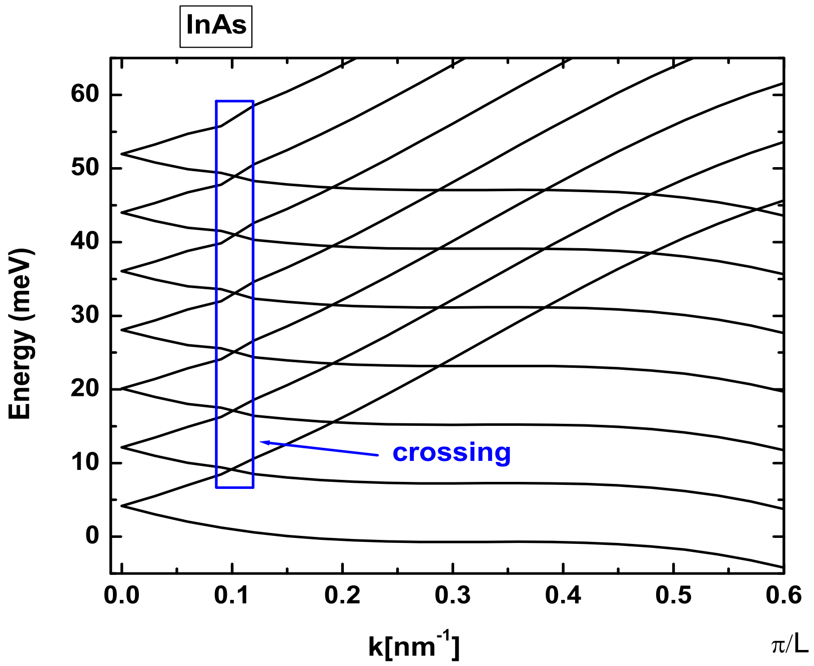

Figure 3: (Color online) Same as Fig. 1(a) but for an InAs nanowire. The level crossing takes place at smaller values of k due to large spin-orbit coupling. Here we chose $\gamma_R = 0.11nm^2$, $\gamma_D = 0.013eVnm^3$.