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# The Influence of Eccentricity on the Crack Breathing in a Rotating Shaft

**L. Rubio, B. Muñoz-Abella, P. Rubio and L. Montero**
**Department of Mechanical Engineering**
**University Carlos III of Madrid, Leganés, Spain**

## Abstract

Machine failures very often occure as a result of the presence and propagation of fatigue cracks caused by the loads and actions that they are subjected to. Those failures sometimes are catastrophic and produce personal injuries or economic problems. The shafts, that perform in bending and torsion, present additional disalignments and, or the unbalances that alter the normal function of the components. When a cracked shaft rotates, the crack opens and closes in what is called the breathing mechanism in which the crack passes from an open state to a close state with a transition between both situations producing a partial opening or closing of the crack. In this paper we present a finite element study of the influence of the eccentricity in a rotating cracked shaft. The model chosen for this study is the classical Jeffcott rotor. To simulate the rotation of the shaft, different angular positions have been considered. The influence of the mass eccentricity in the opening of the crack has been studied considering different angles of ecentricity measured from the position of the crack. In this paper we present the comparison of the partially opening or closing of the crack considering different angles of rotation and different positions of the eccentricity. The work shows the influence of the unbalance of rotating shafts in the crack breathing mechanism and will enable the prediction of the influence of this behaviour on the values of the stress intensity factor and in the propagation of cracks.

**Keywords:** shaft, crack, eccentricity, breathing mechanism, rotative machinery.

# 1 Introduction

The failures of machines are produced quite often by the presence and propagation of fatigue cracks due to the loads and solicitations they carry. Those failures sometimes are catastrophic and produce personal injuries or economic problems. The increasing importance of safety and costs derived from failure in machinery has pushed the researchers in the field of damage detection to analyze the behavior of mechanical components with defects.

Shafts, that are one of the main components of machines, perform in rotation and in bending and torsion. All of them together, can produce the shaft failure by generation and propagation of fatigue cracks.

In the presence of a crack, when a shaft rotates, the crack opens and closes once per revolution. The opening and closing of the crack has been modeled in different ways. The simplest one is to consider that the crack is open or closed, so that the crack is half the rotation in the open state and the other half in the closed one. This model has been used quite often due to the simplicity. However the most feasible behavior of the crack is that called the breathing behavior. In this case, the crack passes from the closed state to the open state gradually in a rotation. It is closed when it is situated in the compression zone of the shaft and it is open when situated in the tensile zone. The transition between both situations produces partial opening or closing of the crack when the static deflection dominates the behavior of the rotating shaft. The partial opening/closing of the crack has been studied, numerically or analytically, by different authors [1, 2, 3] always considering an aligned and balanced shaft. The opening and closing of the crack is very much influenced by the the values taken by the the Stress Intensity Factor (SIF) at the crack front as the shaft rotates. The crack is open while the SIF at the front remains positive, otherwise the crack will be closed.

In the real performance of large shafts it is very usual that the shafts present unbalances or misaligments that modify the normal behavior of the components. A very common unbalance of shafts is due to the presence of eccentric masses [4, 2, 5, 6]. The unbalance, as mention before, modifies the dynamic behavior of the rotating shafts and may hide the presence of the cracks or, on the other hand, can increase the effects of the cracks.

In this paper we present the numerical study of the influence of the eccentricity in a rotating cracked shaft using a finite element model of a cracked Jeffcott rotor. The analysis has been made using the commercial finite element code ABAQUS. We present the comparison, for each angle of rotation, of the partially opening/closing of the crack for different positions of the eccentricity. The work allows to know the influence of the unbalance of rotating shafts in the crack breathing mechanism and allows to predict the influence of this behavior on the values of the stress intensity factor and in the propagation of the cracks.

## 2 The cracked shaft model

The model chosen for this study is the classical Jeffcott rotor widely used in rotordynamics [7, 8, 9, 10, 5, 6]. This simple but useful model consists in a massless shaft simply supported at the ends, with a concentrated mass (a disc). The crack is situated at the mid span of the shaft having a straight front, for sake of simplicity, oriented on a

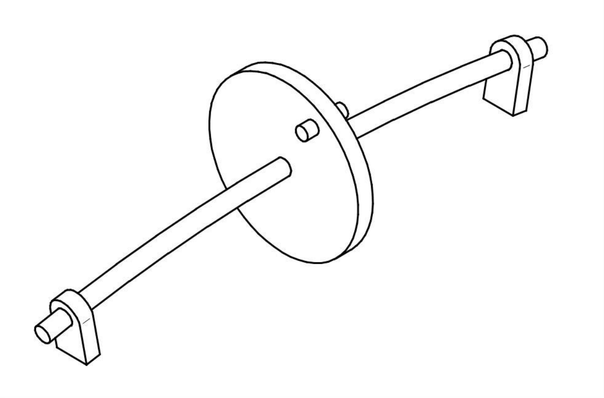

Figure 1: Jeffcott rotor with an eccentric mass.

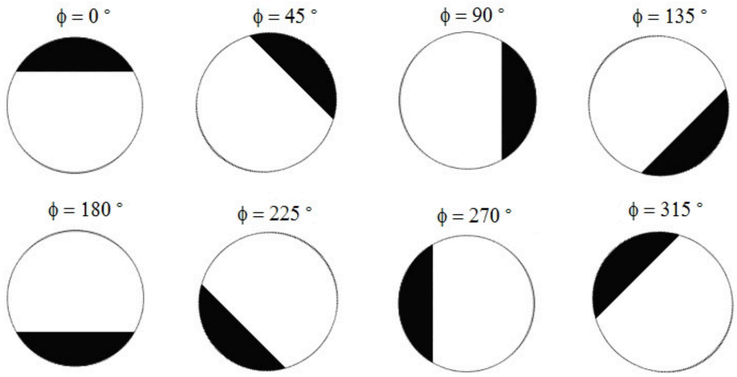


Figure 2: Angular positions of the crack during one rotation

plane normal to the axis of the shaft. The eccentric mass has been placed on the disc of the Jeffcott rotor as an additional mass as can be seen in Figure 1.The round bar total length is equal to 900mm, whereas the diameter is 20mm. The material of the shaft is aluminium with the following mechanical properties: Youngs Modulus $E$=72$GPa$, Poisson ratio $\mu$=0.33 and density $\rho$=2800$Kg/m^3$.

To simulate the rotation of the shaft eight different angular positions, one for every eighth of a rotation has been considered, called angle of rotation $\phi$, see Figure 2. We analyze the static behavior of the shaft (considering the gravity effect) at each angular position. The influence of the mass eccentricity on the opening of the crack has been studied considering that the eccentric mass is placed on the disc and situated at different angles measured from the position of the crack, angle of eccentricity $\theta$, as shown in Figure 3. To simulate the effect of the eccentric mass, an inertial force, $F_e$ corresponding to a mass $m$ located at a distance $e$ from the center of the Jeffcott rotor rotating with the angular rotating velocity $\Omega$ has been included in the analysis, see Figure 4. Values taken in the analysis are $m$=0.2 $Kg$, $e$=80$mm$ and $\Omega$= 1000 $rpm$.

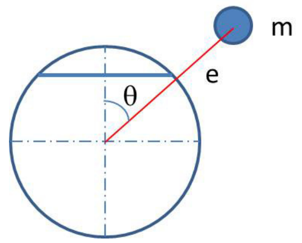


Figure 3: Relative position of the eccentricity

Cracks of three lengths, $\alpha = a/D$, and straight fronts have been modeled to evaluate the influence of both the crack size and the position of the eccentricity in the behavior of the shaft. The numerical simulation of the problem has been carried out using the Finite Element commercial code ABAQUS. For this analysis, the complete 3D model of the shaft has been considered due to there is no symmetry in the evaluated problem. The mesh of the three dimensional model is made employing 8 node linear brick elements. In order to avoid the interpenetration between the crack faces, a surface-to-surface contact interaction has been defined.

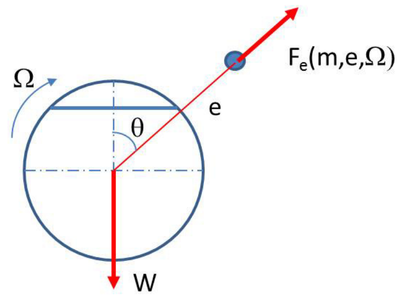


Figure 4: Resume of the applied forces on the shaft

A total of 144 cases have been simulated and analyzed accordingly with the following:

- crack length $\alpha$=0.1; 0.25; 0.5
- eccentricity angle $\theta$= $0^o$; $45^o$; $90^o$; $135^o$; $180^o$
- rotation angle $\phi$= $0^o$; $45^o$; $90^o$; $135^o$; $180^o$; $225^o$; $270^o$; $315^o$; $360^o$

To simulate the balanced shaft, 24 cases corresponding to the same three crack lengths and eight rotation angles have been also modeled in order to compare with the corresponding unbalanced cases.

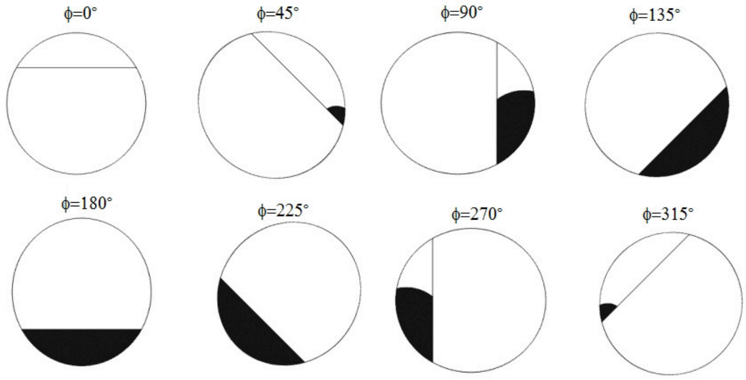


Figure 5: Opening of the crack (in black) for a balanced shaft with $\alpha$=0.25 in a full rotation

# 3 Crack opening

In this section we analyze the results obtained with the numerical simulations proposed previously. Particularly here what is studied is the opening of the cracked zone as the shaft rotates. An example of the results given by the simulations is Figure 5. It shows the representation of the cracked section of a balanced shaft, where the dark zone corresponds to the open part of the crack during a rotation. Each image represents the situation for each rotation angle .

From those images, the open area can be easily measured. In order to compare the results obtained, the percentage of opening area has been derived as (1):

$$\Lambda = \frac{A_o}{A_c} \cdot 100 \tag{1}$$

where $A_o$ and $A_c$ are, respectively, the measured open area for the current case and the total cracked area. This means that $\Lambda$ will take values from 0 to 100, being 0 for the case of a completely closed crack and 100 for fully open cracks.

First of all, the results of the percentage of opening area, $\Lambda$, for a balanced shaft are plotted in Figure 6. Here $\Lambda$ has been plotted versus rotation angle $\phi$ for the case $\alpha$= 0.25. Datum of Figure 6 correspond to those of Figure 5. The extension of this curve to the other crack lengths is shown in Figure 7. As can be seen, while no eccentricity is introduced in the problem, the crack opens and closes as the shaft rotates with symmetry. It is shown also that the opening of the crack es greater proportionally as the crack is longer. These curves will be taken as a reference to analyze the effect of the angle of the eccentricity.

The effect of the eccentricity can be observed in Figure 8. Here the opening of the crack is shown for a selected rotation angle of $\phi = 270^o$ for different eccentricity cases starting with the balanced (no eccentricity) shaft. There is a great difference of opening percentage depending on where the eccentric mass is located with respect to

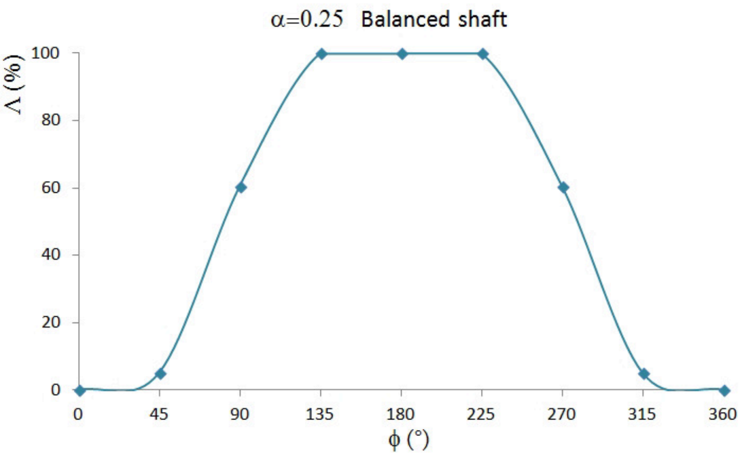


Figure 6: Opening of the crack in a rotation of the balanced shaft

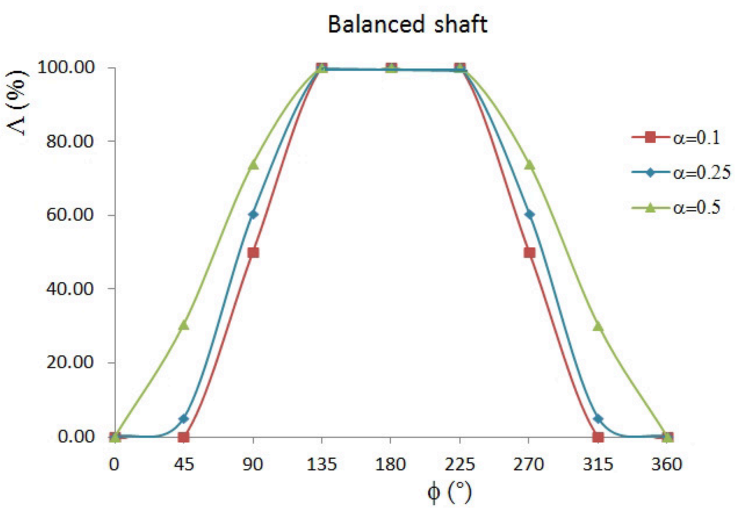


Figure 7: Opening of the crack in a balanced shaft in a rotation for different crack lengths

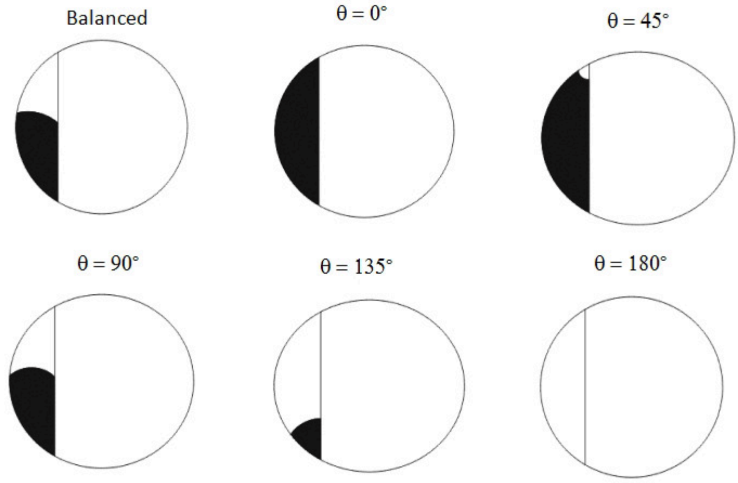


Figure 8: Opening of the crack (in black) for different angles of eccentricity, $\theta$. Case $\phi = 270^o$ and $\alpha$=0.25

the crack. For the selected rotation angle and for the selected $m$, $e$ and $\omega$, the shaft will remain closed (same as if the shaft was uncracked) if the mass is located opposite to the crack, and will be fully open if the mass is in front of the crack, while the balanced shaft will be partially open.

In Figures 9 to 11 the variable angle of eccentricity is introduced. The proportional open area is plotted against the rotation angle for different eccentricity angles for the cases $\alpha$=0.1 to $\alpha$=0.5, respectively.

Looking to the figures we can see that, for the selected $m$, $e$ and $\omega$, if the eccentricity is placed opposite to the crack (eccentricity angle $\theta$=$180^o$), the crack never opens while the crack is small, which means that the cracked shaft behaves as it was an intact shaft. In this situation the crack will never propagate. If the eccentric mass is at the same position as the crack (eccentricity angle $\theta$=0) the crack will be always open. In this situation the crack will be in a good position to propagate. As the eccentric mass gets closer to the crack position (eccentricity angle changing from $180^o$ to $0^o$), the shaft passes from having the crack always closed to having the crack always open. The trend is the same, independently, of the size of the crack.

Another trend that can be observed through the figures is that as the eccentric mass gets closer to the crack position (eccentricity angle changing from $180^o$ to $0^o$), the maximum amount of opening is reached with a small rotation angle. This happens with no dependency of the length of the crack.

As mention before, the percentage of opening increases with the length of the crack. In fact, for a deep crack $\alpha = 0.5$, the crack is partially open most of time of the rotation even for the maximum of the eccentricity angle.

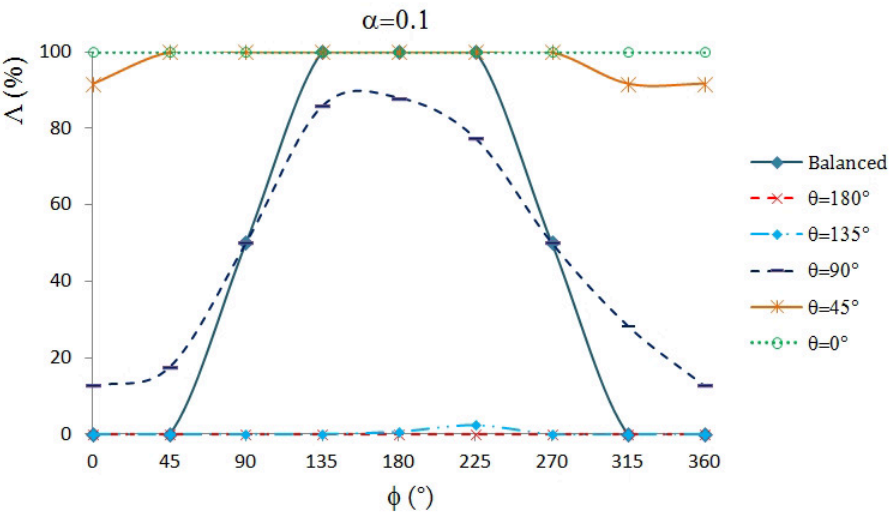


Figure 9: Opening of the crack in a rotation for different eccentricities. $\alpha$=0.1

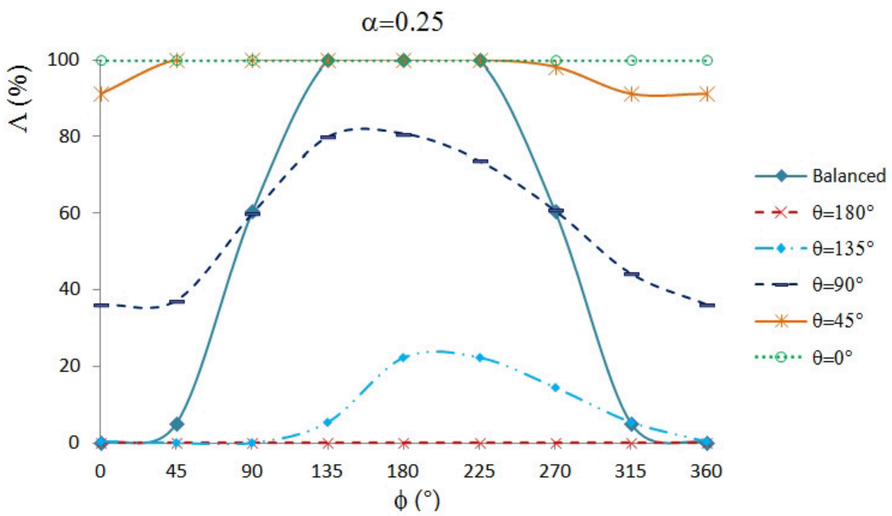


Figure 10: Opening of the crack in a rotation for different eccentricities. $\alpha$=0.25

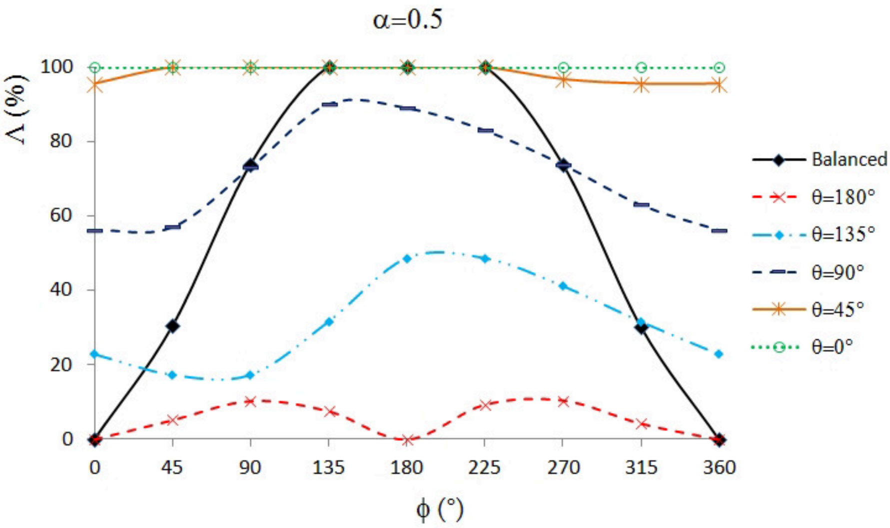


Figure 11: Opening of the crack in a rotation for different eccentricities. $\alpha$=0.5

# 4 Conclusion

A numerical simulation of the rotation of a cracked shaft with a eccentric mass has been carried out. From the numerical results , obtained for a given values of the set $m$, $e$, and $\Omega$, five basic ideas can be extracted:

- if the eccentricity is placed opposite to the crack and the crack is small, the crack never opens so that the crack will never propagate
- if the eccentric mass is in front of the crack, the crack will always be open
- for eccentricity angles between $0^o$ and $180^o$, the opening and closing of the crack will depend on the angle of eccentricity and the crack length
- as the eccentric mass gets closer to the crack position the maximum amount of opening is reached with an smaller rotation angle
- the percentage of opening increases with the length of the crack

The work shows the influence of the unbalance of rotating shafts on the crack breathing mechanism and will allow prediction of the influence of this behaviour on the values of the stress intensity factor and in the propagation of the cracks.

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