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A Hybrid Meta-Heuristic Method and Weighted Least Squares Support Vector Machine Method for the Optimal Shape Design of Gravity Dams

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Abstract

A hybrid meta-heuristic optimization method is presented to efficiently find the optimal shape of concrete gravity dams including the dam-water-foundation rock interaction subject to earthquake loading. The proposed method is based on a hybrid of the gravitational search algorithm (GSA) and particle swarm optimization (PSO), which is called GSA-PSO. In order to reduce the computational cost of dam analysis subject to earthquake loading, a weighted least squares support vector machine (WLS-SVM) is employed to predict the dynamic response of gravity dams. Numerical results demonstrate the high performance of GSA-PSO for optimal shape design of dams.

Keywords: concrete gravity dams, optimal shape, gravitational search algorithm, particle swarm optimization, weighted least squares support vector machine.

1 Introduction

The economy and safety of a concrete gravity dam depend on an appropriate shape design. Hence, a proper shape design of concrete gravity dams is an important problem in dam engineering. The proper shape of dam considering the economy and safety of design, structural considerations, etc. is selected as the final shape. In order to overcome these difficulties and achieve an optimal shape for dams, optimization techniques can be effectively utilized [1].

During the last years, various studies related to design optimization of arch dams were reported [2-9]. In these studies, the effect of arch dam-water-foundation rock interaction was neglected and conventional mathematical models were utilized for analysis approximation and optimization task. In recent years, the studies on the subject of arch dam optimization have been developed. Salajegheh *et al.* [10] employed simultaneous perturbation stochastic approximation (SPSA) method with hydrodynamic effects to find optimum

design. Shape optimization of arch dams using a fuzzy inference system, wavelet neural network and grading strategy is performed by Seyedpoor *et al.* [11, 12]. Akbari and Ahmadi [13] optimized real double curvature parabolic arch dam with the sequential quadratic programming (SQP) method. Akbari *et al.* [1] presented a new algorithm based on Hermit Splines with SQP method for optimal shape design of arch dams. A study was introduced by Seyedpoor *et al.* [14] so that optimal shape design of arch dams including hydrodynamic and the material nonlinearity effects is achieved using an improved particle swarm optimization (IPSO) and a wavelet back propagation (WBP) neural network.

In this study, the optimal shape of arch dams including dam-water-foundation rock interaction is investigated using a hybrid meta-heuristic optimization method. The hybrid meta-heuristic optimization method is based on a combination of gravitational search method (GSA) and particle swarm optimization (PSO), called GSA-PSO. The load cases involved here are gravity load, hydrostatic and hydrodynamic pressures, and earthquake load which is treated with the time history analysis of concrete gravity dam model. The weight of concrete gravity dam body is considered as the objective function. The design constraints include behaviour, geometric and stability constraints. The design variables are the shape parameters of the concrete gravity dam. Due to the fact that the computational cost of optimization in time history analysis is high, weighted least squares support vector machine (WLS-SVM) is employed to predict the time history response of dams. Therefore, WLS-SVM can substantially reduce the computing effort of optimization procedure. In order to assess the efficiency of the proposed optimization method, the optimization task is performed by GSA, PSO and GSA-PSO algorithms. Numerical results show the computational advantages of the proposed GSA-PSO for optimal shape design of concrete gravity dams.

2 Gravity dam optimization problem

The gravity dam optimization problem subjected to earthquake loading is stated as follows:

Minimize
$$f(X)$$

Subject to $g_j(X,t) \le 0$ $j = 1, 2, ..., m$ (1)
 $X^L \le X \le X^U$

where f, g_j and t are the objective function, the constraints and the time. X^L and X^U are the lower bound and the upper bound of the design variables, X.

In order to define the geometrical model of gravity dams, the shape of gravity dams can be defined using seven parameters. According to the model shown in Figure 1, a gravity dam can be created by a vector X that has seven components including shape parameters of the concrete gravity dam as:

$$X = \{ b \ b_1 \ b_2 \ b_3 \ H_2 \ H_4 \ H_5 \}$$
(2)

where b and H_1 are two parameters required to defined crest and free board of gravity dam, respectively. H_3 depends on H_4 and reservoir water level (H). The shape parameters of the gravity dam are shown in Figure 1.



Figure 1: Geometrical model of gravity dam.

In the optimization problem, f(X) represents the concrete weight of a gravity dam body that should be minimized. The concrete weight can be determined by:

$$f(X) = W = \rho_c gV \tag{3}$$

where ρ_c and V are mass density and volume of gravity dam, respectively. g is gravity acceleration.

Also, in Equation (1) $g_{j}(X,t)$ are inequality constraints that may be categorized into the behaviour, geometric and stability constraints. In this study, the principal stresses of each node for the gravity dam is considered as the behaviour constraints, which must be satisfied for all time points of the earthquake interval as follows:

$$g_{k,1} = \frac{\sigma_1(X,t)}{\sigma^+} - 1 \qquad k = 1, 2, \dots, nj$$
(4)

$$g_{k,2} = \frac{\left|\sigma_{2}(X,t)\right|}{\sigma^{-}} - 1 \quad k = 1, 2, ..., nj$$
(5)

where $\sigma_1(\cdot)$ and $\sigma_2(\cdot)$ are the principal stresses of each node in time *t*, respectively. σ^+ and σ^- are the allowable tensional and compressive stresses. *nj* is the total number of dam nodes which created by meshing of the dam body.

The stability of a dam is ordinarily expressed in terms of its factors of safety against sliding and overturning, respectively. The factor of safety against sliding is simply the ratio of the total frictional force, F_V , which the foundation can develop to the force tending, F_H , to cause sliding as follows [15]:

$$g_s = \frac{\sum F_V}{\sum F_H} - 1.75 \tag{6}$$

The factor of safety against overturning about the toe is the ratio of the resisting moments, M_R , to the overturning moments, M_O , which is considered as stability constraint. It is given as [15]:

$$g_o = \frac{\sum M_R}{\sum M_O} - I \tag{7}$$

In this study, the penalty function is used to deal with constrained search spaces as:

$$fit(\mathbf{X}) = \begin{cases} f(\mathbf{X}) & \text{if } \mathbf{X} \in \mathbb{R}^d \\ f(\mathbf{X})(1+r_p \ G(\mathbf{X},t)) & \text{otherwise} \end{cases}$$
(8)

where fit(X) and r_p is modified function (fitness function) and an adjusting coefficient; R^d denotes the feasible search space; and G(X,t) is the penalization factor, which is defined as the sum of all active constraints violations; as indicated:

$$G(X,t) = \sum_{t=0}^{T} \sum_{k=1}^{n_j} \left[\max(g_{k,1}, 0)^2 + \max(g_{k,2}, 0)^2 \right] + \max(g_s, 0)^2 + \max(g_s, 0)^2 \quad (9)$$

3 Finite element model of gravity dam-water-foundation rock system

In solving the fluid-structure interaction problem using finite element method (FEM), the discretized dynamic equations of the fluid and structure need to be considered simultaneously to obtain the coupled fluid-structure equation.

3.1 Structural responses

The solid dam is discretized by using FEM and its equations of seismic motion including the effects of the reservoir and the foundation are expressed as [16]:

$$\boldsymbol{M}_{s}\boldsymbol{\ddot{u}}_{e} + \boldsymbol{C}_{s}\boldsymbol{\dot{u}}_{e} + \boldsymbol{K}_{s}\boldsymbol{u}_{e} = -\boldsymbol{M}_{s}\boldsymbol{\ddot{u}}_{g} + \boldsymbol{Q}\boldsymbol{p}_{e}$$
(10)

where M_s is the structural mass matrix, C_s is the structural damping matrix, K_s is the structural stiffness matrix, u_e is the vector of the nodal displacements relative to the ground, \ddot{u}_g is the vector of the ground acceleration, Qp_e represents the nodal force vector associated with the hydrodynamic pressures produced by the reservoir.

The structural damping in the system is usually included by using a Rayleigh type of damping matrix given by [16]:

$$C_{s} = \overline{\alpha} \, \boldsymbol{M}_{s} + \overline{\beta} \, \boldsymbol{K}_{s} \tag{11}$$

where $\overline{\alpha}$ and $\overline{\beta}$ are constants adjusted to obtain a desirable damping in the system, usually on the basis of given modal damping ratios.

3.2 Reservoir responses

For a compressible and inviscid fluid, the hydrodynamic pressure, p, resulting from the ground motion of the rigid dam satisfies the wave equation in the form [17, 18]:

$$\nabla^2 p = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} \tag{12}$$

where c is the velocity of sound in water and ∇^2 is the Laplacian operator in two dimensions.

The following boundary conditions are defined by assuming that the effects of surface waves and the viscosity of the fluid are neglected. As shown in Figure 2, some boundary conditions may be imposed on the fluid domain as follows:



Figure 2: The boundary conditions of the fluid domain [18].

i) At the fluid-solid interface (S1),

$$\frac{\partial p}{\partial n} = -\rho_w a_n \tag{13}$$

where *n* is the unit normal vector, a_n is the normal acceleration on the interface and ρ is the mass density of the fluid.

ii) At the bottom of the fluid domain (S2),

$$\frac{\partial p}{\partial n} = -\rho \, a_n - \overline{q} \, \frac{\partial p}{\partial t} \tag{14}$$

where \overline{q} is a damping coefficient which is the fundamental parameter characterizing the effects of the reservoir bottom materials. Relation between damping coefficient and ratio of reflected hydrodynamic pressure wave, α , is [19]:

$$\alpha = \frac{l - \overline{q}c}{l + \overline{q}c} \tag{15}$$

iii) At the far end (S3); A Sommerfield-type radiation boundary condition can be implemented, namely:

$$\frac{\partial p}{\partial n} = -\frac{\dot{p}}{c} \tag{16}$$

iv) At the free surface (S4):

$$p = 0 \tag{17}$$

Equations (12)-(17) can be discretized to get the matrix form of the wave equation as [16, 17]:

$$\boldsymbol{M}_{f} \boldsymbol{\ddot{p}}_{e} + \boldsymbol{C}_{f} \boldsymbol{\dot{p}}_{e} + \boldsymbol{K}_{f} \boldsymbol{p}_{e} + \boldsymbol{\rho}_{w} \boldsymbol{Q}^{T} (\boldsymbol{\ddot{u}}_{e} + \boldsymbol{\ddot{u}}_{g}) = 0$$
(18)

where M_f , C_f and K_f are the fluid mass, damping and stiffness matrices, respectively, and p_e and \ddot{u}_e are the nodal pressure and relative nodal acceleration vectors, respectively. The term $\rho_w Q^T$ is also often referred to as coupling matrix.

3.3 The coupled fluid-structure equation

Equations (10) and (18) describe the complete finite element discretized equations for the gravity dam-water-foundation rock interaction problem and can be written in an assembled form as [17, 18]:

$$\begin{bmatrix} \boldsymbol{M}_{s} & \boldsymbol{0} \\ \boldsymbol{M}_{fs} & \boldsymbol{M}_{f} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{u}}_{e} \\ \ddot{\boldsymbol{P}}_{e} \end{bmatrix} + \begin{bmatrix} \boldsymbol{C}_{s} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{C}_{f} \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{u}}_{e} \\ \dot{\boldsymbol{p}}_{e} \end{bmatrix} + \begin{bmatrix} \boldsymbol{K}_{s} & \boldsymbol{K}_{fs} \\ \boldsymbol{0} & \boldsymbol{K}_{f} \end{bmatrix} \begin{bmatrix} \boldsymbol{u}_{e} \\ \boldsymbol{p}_{e} \end{bmatrix} = \begin{bmatrix} -\boldsymbol{M}_{s} \ddot{\boldsymbol{u}}_{g} \\ -\boldsymbol{M}_{fs} \ddot{\boldsymbol{u}}_{g} \end{bmatrix}$$
(19)

where $\boldsymbol{M}_{fs} = \boldsymbol{\rho}_{w} \boldsymbol{Q}^{T}$ and $\boldsymbol{K}_{fs} = -\boldsymbol{Q}$. Equation (19) expresses a second order linear differential equation having unsymmetrical matrices and may be solved by means of direct integration methods.

In the present study, the finite-element idealization of gravity dam-waterfoundation rock system is implemented on the mentioned theory and assumptions. The gravity dam is treated as a two-dimensional-linear structure. The four-node solid element is utilized to mesh the dam body. The four-node fluid element is used to discretize the fluid medium and the interface of the fluid-structure interaction problem. The element has four degrees of freedom per node: translations in the nodal x, y and x directions and pressure. The translations are applicable only at nodes that are on the interface. Interaction between the fluid and foundation rock is considered through a damping boundary condition applied along the bottom and sides of the reservoir defined by the second term of Equation (14). The damping matrix of the dam is also accomplished using the Rayleigh damping taken into account by Equation (11). In this study, foundation rock treating as a linearly elastic structure is represented via a four-nodded solid element. Also, only the effects of foundation flexibility are considered and the inertia and damping effects of the foundation rock are neglected. In the analysis phase, a static analysis of the gravity dam-water-foundation rock system is initially implemented under a gravity load and a hydrostatic pressure and then, the linear dynamic analysis of the system is performed using the Newmark time integration method [16]. After that, nodal relative displacement vector of the structure is utilized to evaluate the principal stresses at the centre of dam elements via conventional FEM.

4 Meta-heuristic optimization method

In this study, to find the optimal shape of gravity dams a hybrid meta-heuristic optimization algorithm is proposed. This meta-heuristic optimization method is based on a combination of gravitational search method (GSA) and particle swarm optimization (PSO), called GSA-PSO.

4.1 Particle swarm optimization

Recently, in structural engineering the successful applications of PSO as an optimization engine have been reported by Refs. [20-26]. PSO has been inspired by the social behaviour of animals such as fish schooling, insects swarming and birds flocking. The PSO algorithm is introduced by Kennedy and Eberhart [27] in the mid 1990s, while attempting to simulate the graceful motion of bird swarms as a part of a socio-cognitive study. It involves a number of particles, which are initialized randomly in the search space of an objective function. These particles are referred to as swarm. Each particle of the swarm represents a potential solution of the optimization problem.

The *i*th particle in *l*th iteration is associated with a position vector, X_i^l , and a velocity vector, V_i^l , that shown as following:

$$\begin{aligned} \boldsymbol{X}_{i}^{l} &= \{ x_{i1}^{l}, x_{i2}^{l}, \dots, x_{iD}^{l} \} \\ \boldsymbol{V}_{i}^{l} &= \{ v_{i1}^{l}, v_{i2}^{l}, \dots, v_{iD}^{l} \} \end{aligned}$$
(20)

where D is dimension of the solution space.

The particle fly through the solution space and its position is updated based on its velocity, the best position particle (*pbest*) and the global best position (*gbest*) that swarm has visited since the first iteration as,

$$V_{i}^{l+1} = \omega^{l} V_{i}^{l} + c_{1} r_{1} (pbest_{i}^{l} - X_{i}^{l}) + c_{2} r_{2} (gbest^{l} - X_{i}^{l})$$
(21)

$$X_i^{l+1} = X_i^l + V_i^{l+1}$$
(22)

where r_1 and r_2 are two uniform random sequences generated from interval [0, 1]; c_1 and c_2 are the cognitive and social scaling parameters, respectively and ω^l is the inertia weight used to discount the previous velocity of particle preserved.

Shi and Eberhart [28] proposed that the cognitive and social scaling parameters c_1 and c_2 to be selected such that $c_1=c_2=2$ to allow the product c_1r_1 or c_2r_2 to have a mean of 1. One of the main parameters affecting the performance of PSO is the inertia weight (ω) in achieving efficient search behaviour.

A dynamic variation of inertia weight is proposed by linearly decreasing ω with each iteration algorithm as shown in Equation (23) [28]:

$$\omega = \omega_{\max} - \frac{\omega_{\max} - \omega_{\min}}{l_{\max}}l$$
(23)

where ω_{\max} and ω_{\min} are the maximum and minimum values of ω , respectively. Also, l_{max} is the numbers of maximum iteration.

4.2 Gravitational search algorithm

The gravitational search algorithm (GSA) was first introduced by Rashedi *et al.* [29] as a new stochastic population-based heuristic optimization method. This approach provides an iterative method that simulates mass interactions, and moves through a multi-dimensional search space under the influence of gravitation. This heuristic algorithm has been inspired by the Newtonian laws of gravity and motion. In GSA, agents are considered as objects and their performance are measured by their masses, and all these objects attract each other by the gravity force, while this force causes a global movement of all objects towards the objects with heavier masses.

In GSA, each agent of the population represents a potential solution of the optimization problem. The *i*th agent in *l*th iteration is associated with a position vector, $X_i(l)$, and a velocity vector, $V_i(l)$, that shown as following:

$$X_{i}(l) = \{x_{i}^{1}, x_{i}^{2}, ..., x_{i}^{D}\}$$

$$V_{i}(l) = \{v_{i}^{1}, v_{i}^{2}, ..., v_{i}^{D}\}$$
(24)

The mass of each agent is calculated after computing the current population fitness for a minimization problem, as follows [29]:

$$q_i(l) = \frac{fit_i(l) - worst(l)}{best(l) - worst(l)}$$
(25)

$$M_{i}(l) = \frac{q_{i}(l)}{\sum_{i=1}^{N} q_{i}(l)}$$
(26)

where N, $M_i(l)$ and $fit_i(l)$ represent the population size, the mass and the fitness value of agent *i* at *l*the iteration, and worst(l) and best(l) are defined as follows:

$$best(l) = \min\{fit_1(l), fit_2(l), ..., fit_N(l)\}$$
(27)

$$worst(l) = \max\{fit_1(l), fit_2(l), ..., fit_N(l)\}$$
 (28)

To compute the acceleration of an agent, total forces from a set of heavier masses applied on it should be considered based on a combination of the law of gravity and the second law of Newton on motion (Equation (29)) [29]. Afterwards, the next velocity of an agent is calculated as a fraction of its current velocity added to its acceleration (Equation (30)). Then, its position could be calculated using Equation (31).

$$a_i^d(l) = \sum_{j \in kbest, \ j \neq i} rand_j G(l) \frac{M_j(l)}{R_{ij}(l) + \varepsilon} (x_j^d(l) - x_i^d(l))$$
(29)

$$v_i^d(l+1) = rand_i v_i^d(l) + a_i^d(l)$$
(30)

$$x_i^d (l+1) = x_i^d (l) + v_i^d (l+1)$$
(31)

where a_i^d , v_i^d and x_i^d present the acceleration, velocity and position of *i*th agent in dimension *d*, respectively. *rand*_i and *rand*_j are two uniform random numbers in the interval [0, 1]; ε is a small value; *n* is the dimension of the search space; and $R_{ij}(l)$ is the Euclidean distance between agent *i* and *j*. *kbest* is the set of first *K* agents with the best fitness value and biggest mass, which is a function of time, initialized to K_0 at the beginning and decreased with time. *G* is a decreasing

function of time. In this study, G(l) is considered as a linear decreasing function in the GSA algorithm as follows:

$$G(l) = G_0 (1 - \frac{l}{l_{\text{max}}})$$
(32)

This function is set to G_0 at the beginning and decreases exponentially towards zero with lapse of time.

4.3 The proposed meta-heuristic optimization method

The PSO algorithm is one of the most proficient meta-heuristic techniques which is an algorithm inspired by the social behaviour of flock populations. In structural engineering the shortcoming of the PSO algorithm is to need a great number of function evaluations for finding the global solution. It is expected that this drawback can be dealt with selecting an adequate initial swarm. In this study, the GSA algorithm as local search finds an optimal initial swarm for commencing the PSO algorithm. The GSA-PSO method is executed the following steps:

- Step 1: Employ the GSA algorithm as local search and find the optimum design, X_{GSA} .
- Step 2: Copy N_{GSA} times to create the some part of the initial swarm for PSO.
- Step 3: Select other particles of the initial swarm, $X_{rnd, j}$ $(j = 1, 2, ..., N_{PSO} N_{GSA})$, randomly.
- Step 4: Utilize PSO to find a more accurate optimum design using the optimal initial swarm.

5 Weighted least squares support vector machines (WLS-SVM)

Recently, support vector machines (SVMs) have been successfully used as an excellent machine learning algorithm [30]. The SVM is developed based on the structural risk minimization (SRM), which can escape from several drawbacks, such as local minimum and the necessity of a large number of controlling parameters in artificial neural networks (ANNs). The weighted least squares support vector machines (WLS-SVM) is introduced by Suykens *et al.* [30] to decrease the training computational effort of SVM in the large-scale problem, and be better than that of least squares version of SVM (LSSVM) [30, 31].

The WLS-SVM is described as the following optimization problem in primal weight space [30]:

$$\min J(\boldsymbol{w}, \boldsymbol{e}) = \frac{1}{2} \left\| \boldsymbol{w} \right\|^2 + \frac{1}{2} \gamma \sum_{i=1}^n \overline{v}_i e_i^2$$
(33)

subject to the following equality constraints:

$$y_i = \mathbf{w}^T \varphi(\mathbf{x}_i) + b + e_i, \quad i = 1, 2, ..., n$$
 (34)

with $\{x_i, y_i\}_{i=1}^n$ a training data set, input data $x_i \in R^n$ and output data $y_i \in R$. $\varphi(.): R^n \to R^d$ is a function which maps the input space into a higher dimensional space. The vector $w \in R^d$ represents weight vector in primal weight space. The symbols $e_i \in R$ and $b \in R$ represent error variable and bias term, respectively.

By the optimization problem (33) and the training set, the model of WLS-SVM is defined as follows:

$$y(\mathbf{x}) = \mathbf{w}^T \varphi(\mathbf{x}) + b \tag{35}$$

It is impossible to indirectly calculate w from (33), because of the structure of the function $\varphi(x)$ is unknown in general. Therefore, the dual problem shown in Equation (33) is minimized by the Lagrange multiplier method as follows:

$$L(\boldsymbol{w}, \boldsymbol{b}, \boldsymbol{e}; \boldsymbol{x}) = J(\boldsymbol{w}, \boldsymbol{e}) - \sum_{i=1}^{n} \alpha_{i} (\boldsymbol{w}^{T} \varphi(\boldsymbol{x}_{i}) + \boldsymbol{b} + \boldsymbol{e}_{i} - \boldsymbol{y}_{i})$$
(36)

According to the Karush-Khun-Tucker (KKT) conditions, eliminating w and e the solution is given by the following set of linear equation:

$$\begin{bmatrix} \Omega + V_{\gamma} & I_{n}^{T} \\ I_{n} & \theta \end{bmatrix} \begin{bmatrix} \alpha \\ b \end{bmatrix} = \begin{bmatrix} y \\ 0 \end{bmatrix}$$
(37)

where

$$\boldsymbol{V}_{\gamma} = diag\{1/\gamma \overline{v}_{1},...,1/\gamma \overline{v}_{n}\}; \boldsymbol{\Omega}_{i,j} = \langle \varphi(\boldsymbol{x}_{i}), \varphi(\boldsymbol{x}_{j}) \rangle_{H} \quad i, j = 1,...,n$$

$$\boldsymbol{y} = [\boldsymbol{y}_{1},...,\boldsymbol{y}_{n}]^{T}; \boldsymbol{I}_{n}^{T} = [1,...,1]; \boldsymbol{\alpha} = [\alpha_{1},...,\alpha_{n}]$$
(38)

According to Mercer's condition, a kernel K(.,.) is selected, such that:

$$K(\boldsymbol{x}, \overline{\boldsymbol{x}}) = \left\langle \varphi(\boldsymbol{x}), \varphi(\overline{\boldsymbol{x}}) \right\rangle_{H}$$
(39)

So, the resulting WLS-SVM model for the prediction of functions becomes:

$$y(\mathbf{x}) = \sum_{i=1}^{n} \alpha_i K(\mathbf{x}_i, \mathbf{x}) + b$$
(40)

In this study, radial basis function (RBF) is selected as the kernel function of WLS-SVM. The kernel function is defined as follows:

$$K(\boldsymbol{x}_i, \boldsymbol{x}_j) = \exp(-\frac{\|\boldsymbol{x}_i - \boldsymbol{x}_j\|^2}{2\sigma^2})$$
(41)

where σ is positive real constant, and is usually so-called kernel width.

6 Test example

In order to investigate the computational efficiency of the hybrid meta-heuristic optimization method for the shape optimization of concrete gravity dams, Pine flat dam is considered as a real-world structure. This dam located on King's River near Fresno, California. The dam structure is 400 *ft* height with a crest length of 1840 *ft* and its construction about 9491.94 *kip* concrete. The detailed properties of the damwater system have been provided in Refs. [32, 33]. The ground motion recorded at Taft Lincoln School Tunnel during Kern country, California, earthquake of July 21, 1952, is selected as the excitation for analyses of Pine Flat dam [32, 33]. The ground motion acting transverse to the axis of the dam is defined by the S69E component of the recorded motion. This component of the recorded ground motion is shown in Figure 3.



Figure 3: Ground motion at Taft Lincoln Tunnel; Kern country, California, 1952.

In the present study, to create the gravity dam geometry, seven design variables are considered. The lower and upper bounds of design variables required for the optimization process can be determined using some preliminary design methods [34]:

Material	Property	Value	
	Modulus of elasticity	3.25×10^6 psi	
Concrete	Poisson's ratio	0.2	
	Mass density	155 lb/ft ³	
	Mass density	62.4 lb/ft ³	
Water	Wave velocity	4720 ft/sec	
	Wave reflection coefficient	0.817	
Foundation	Modulus of elasticity	10 ⁷ psi	
	Poisson's ratio	0.33	

The properties of concrete, water and foundation are shown in Table 1.

Table 1: The properties of materials.

The parameters of PSO, GSA and GSA-PSO used in optimization process are given in Table 2. The optimization task is performed by a core[™]2 Duo 2 GHz CPU and all computing times are evaluated by clock time.

Method	Parameter	Value
	Population size	
PSO	Total iteration number	
	$arnothing_{\min}$	
	$\omega_{ m max}$	0.9
	Population size	50
GSA	Total iteration number	100
	G_0	50.0
	The initial number of particles generated via GSA	20
GSA-PSO	The initial number of particles generated on random basis	
	The maximum number of iterations performing by GSA	50
	The maximum number of iterations performing by PSO	50

Table 2. The parameters of PSO, GSA and GSA-PSO methods.

For design optimization of the selected dam two cases related to various conditions of dam-water-foundation rock interaction problem are considered as follows:

Case 1: Dam with full reservoir and rigid foundation.

Case 2: Dam with full reservoir and flexible foundation.

6.1 Finite element model of Pine Flat gravity dam

An idealized model of Pine Flat gravity dam-water-foundation rock system is simulated using FEM as shown in Figure 4. The geometric properties of the dam can be found in Ref. [33].



Figure 4: Finite element model of Pine Flat dam-water-foundation rock system.

In order to validate FEM with the employed assumptions, the first natural frequency of the symmetric mode of the gravity dam for four cases are determined from the frequency response function for the crest displacement. The results are compared with those reported by Chopra and Chakrabarti [33] as given in Table 3.

Foundation			Natural frequency (Hz)			
Case	rock condition	Water	Chopra and Chakrabarti [33]	The present work	Error (%)	
1	Rigid	Empty	3.1546	3.152	0.082	
2	Rigid	Full	2.5189	2.525	0.242	
3	Flexible	Empty	2.9325	2.93	0.085	
4	Flexible	Full	2.3310	2.383	2.18	

Table 3: A comparison of the natural frequencies from the literature with FEM.

It can be observed that a good conformity has been achieved between the results of the present work with those reported in the literature.

6.2 Data selection, training and testing WLS-SVM

To generate a database for training and testing WLS-SVM, design variable vector of the gravity dam defined in (2) is considered as the input vector of WLS-SVM, and the sum of all active constraints violations given by (9) is taken as the output of WLS-SVM. In this study, Latin Hypercube Design (LHD) proposed for computer experiments by McKay *et al.* [35] is used for generating 200 combinations of the design variables. The effective responses of all gravity dam samples are determined using finite element analysis (FEA). The samples are selected on a random basis and from which 150 and 50 ones are employed to train and test the performance generality of WLS-SVM. A mean absolute percentage error, *MAPE*, relative root-mean-squared error, *RRMSE*, and the absolute fraction of variance, R^2 , are used to evaluate the performance of the WLS-SVM model as follows:

$$MAPE = \frac{1}{n_t} \sum_{i=1}^{n_t} 100 \times \left| \frac{y_i - \overline{y}_i}{y_i} \right|$$
(43)

$$RRMSE = \sqrt{\frac{n_{t} \sum_{i=1}^{n_{t}} (y_{i} - \overline{y}_{i})^{2}}{(n_{t} - 1) \sum_{i=1}^{n_{t}} y_{i}^{2}}}$$
(44)

$$R^{2} = 1 - \left(\frac{\sum_{i=1}^{n_{i}} (y_{i} - \overline{y}_{i})^{2}}{\sum_{i=1}^{n_{i}} \overline{y}_{i}^{2}}\right)$$
(45)

where y and \overline{y} are actual value and predicted value, respectively; and n_t is the number of testing samples. The smaller *RRMSE* and *MAPE* and the larger R^2 are indicative of better performance generality.

In this study, the 10-fold CV is used for finding the parameters and training WLS-SVM. The statistical values and the optimal values of γ and σ for predicting the effective responses of dam samples found from testing modes are obtained and the results are listed in Table 4.

Case	Optimal values		Statistical values		
	γ	σ	MAPE	RRMSE	R^2
1	1364.151	80.397	2.4	0.0292	0.9997
2	489.439	32.856	3.62	0.0305	0.9992

Table 4: Statistical values for WLS-SVM in testing mode

6.3 Result of optimization

In order to consider the stochastic nature of the optimization process, ten independent optimization runs are performed for each design case and the best solutions are reported. The optimum designs of the gravity dam for two cases using GSA, PSO and the proposed GSA-PSO based on WLS-SVM as approximate analysis are given in Tables 5 and 6.

Variables	Optimization Method			
variables	PSO	GSA	GSA-PSO	
b (<i>ft</i>)	25.3849	21.5092	19.6700	
$b_1(ft)$	31.4449	31.7477	32.0582	
$b_2(ft)$	29.5940	30.1002	29.2068	
$b_3(ft)$	212.4020	219.4519	216.8624	
$H_2(ft)$	15.4000	12.6024	12.6000	
$H_4(ft)$	341.6600	325.4956	331.1774	
$H_5(ft)$	270.0000	270.0000	270.0000	
Concrete weight (kip)	8309.34	8216.27	8025.79	
Data generation time (min)	279	279	279	
Time of training WLS-SVM (min)	1.04	1.04	1.04	
Optimization time (min)	0.48	2.69	3.09	
Overall computing time (min)	280.52	282.73	283.13	
Required approximate analyses	3800	3350	3100	

Table 5: Optimum designs of the gravity dam for case 1.

Variables	Optimization Method			
variables	PSO	GSA	GSA-PSO	
b (<i>ft</i>)	24.6921	26.4482	21.1757	
$b_1(ft)$	31.1698	32.2928	32.2166	
$b_2(ft)$	31.0539	30.6843	30.5364	
$b_3(ft)$	223.7499	226.7058	239.4790	
$H_2(ft)$	15.3994	12.9876	13.7315	
$H_4(ft)$	341.6600	336.7767	337.5632	
$H_5(ft)$	280.5927	270.0000	270.0000	
Concrete weight (kip)	8777.68	8744.28	8675.56	
Data generation time (min)	327.56	327.56	327.56	
Time of training WLS-SVM (min)	1.04	1.04	1.04	
Optimization time (min)	0.58	2.77	3.12	
Overall computing time (min)	329.18	331.37	331.72	
Required approximate analyses	4000	3500	3150	

Table 6: Optimum designs of the gravity dam for case 2.

The optimal dams are also analyzed by an accurate FEA, and the value of maximum violated constraint is zeros for all design cases. Therefore, the optimum solutions are proper designs. In GSA-PSO the maximum numbers of searches (i.e. number of approximate structural analyses) for two cases are set equal to 3100 and 3150, respectively. Therefore, the numerical results show that the GSA-PSO converges to superior solutions to GSA and PSO while needing much lower function evaluations. The convergence histories of the best solutions of PSO, GSA and GSA-PSO for two cases are shown in Figures 5 and 6.



Figure 5: Convergence histories of the best solution of PSO, GSA and GSA-PSO for case 1.



Figure 6: Convergence histories of the best solution of PSO, GSA and GSA-PSO for case 2.

7 Conclusion

A hybrid meta-heuristic optimization method is presented to find the optimal shapes of concrete gravity dams including dam-water-foundation rock interaction subject to earthquake loading. The optimization method is based on a hybrid of the gravitational search algorithm (GSA) and particle swarm optimization (PSO), which is called the GSA-PSO. The main idea behind the proposed GSA-PSO is to combine the advantages and avoid the disadvantages of the GSA and PSO methods. The GSA-PSO method can increase the probability of finding the global optimum while requiring a lower structural analysis. In order to approximate the dynamic analysis of the gravity dam instead of directly performing it by a time consuming finite element analysis (FEA), the weighted least squares support vector machine (WLS-SVM) is utilized. Numerical results show that the proper optimal design using GSA-PSO can be achieved for the gravity dam. The optimum designs of the proposed GSA-PSO are also compared with those produced by GSA and PSO. GSA-PSO shows the improvement in terms of computational efficiency, optimum solution, the number of function measurements and convergence history in the optimization process.

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