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Paper 111

Linearization of the Hoek-Brown Failure Criterion for Non-Hydrostatic Stress Fields

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Abstract

In this paper, a novel approximate method is proposed for the linearization of the Hoek-Brown (H-B) plastic potential function to circumvent some computational difficulties encountered in the analysis of underground openings in dilatant rock masses subject to non-hydrostatic in situ stresses. This is achieved by computing the equivalent Mohr-Coulomb (M-C) dilation parameter. The proposed method is validated by conducting elastoplastic and elastic-brittle-plastic plane strain finite element analysis of circular and non-circular openings in dilatant and non-dilatant rock masses subject to hydrostatic and non-hydrostatic in situ stresses. Displacements computed by using the H-B strength and dilation parameters are compared with those obtained by using the equivalent M-C parameters. The agreement in results is found to be very good.

Keywords: Hoek-Brown failure criterion, plasticity, elastic-brittle-plastic rock, finite element analysis, underground openings, dilatancy.

1 Introduction

The design of underground openings in rock requires the computation of stresses and displacements around the openings. The linear Mohr-Coulomb (M-C) failure criterion is conventionally used for the analysis of problems in geotechnical engineering. Figure 1 shows the failure envelope in the principal stresses space for this failure criterion expressed as

$$\sigma_1 = \sigma_c + K\sigma_3 \quad (1a)$$

$$K = \tan\chi = \frac{1+\sin\phi}{1-\sin\phi} \quad (1b)$$

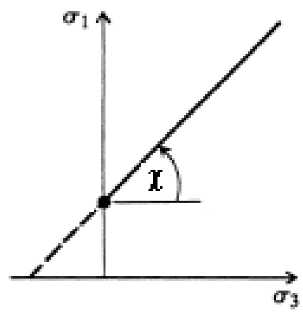


Figure 1: Mohr-Coulomb failure envelope in the principal stresses space

where σ_1 and σ_3 are the major and minor principal stresses, respectively; σ_c is the uniaxial compressive strength of rock; and ϕ is the angle of internal friction. σ_c may be expressed in terms of the M-C strength parameters ϕ and cohesion c as

$$\sigma_c = \frac{2c\cos\phi}{1-\sin\phi} \tag{2}$$

The M-C failure criterion is known to be unsuitable for many types of rock, particularly, discontinuous rock masses. For such rocks, the non-linear Hoek-Brown (H-B) failure criterion is more suitable [1, 2]. Figure 2 shows the failure envelope in the principal stresses space for this failure criterion expressed as

$$\sigma_1 = \sigma_3 + \sigma_c\left(m_b\frac{\sigma_3}{\sigma_c} + s\right)^a \tag{3}$$

where m_b, s and a are the H-B constants [1] before yielding.

Most of the computational tools in geotechnical engineering have not implemented the H-B failure criterion. One of the reasons for this may be that some computational difficulties arise in the use of this failure criterion due to its non-linearity [3, 4]. For the elastoplastic or elastic-brittle-plastic analysis, the results may not converge to the correct solution, particularly for a dilatant rock mass.

In order to circumvent the above mentioned computational difficulties, several approaches [5, 6, 7, 8, 9, 10, 11] have been proposed to linearize the H-B failure criterion by obtaining equivalent M-C (EM-C) strength parameters. All of these approaches were based on a linear plastic potential function Q corresponding to the M-C failure criterion given by

$$Q = \sigma_1 - K_d\sigma_3 \tag{4a}$$

$$K_d = \frac{1+\sin\psi}{1-\sin\psi} \tag{4b}$$

where ψ is the angle of dilation, assumed to be constant. However, it is known [12] that the constant dilation is impractical, as dilation depends on the plastic strain and the confining stress.

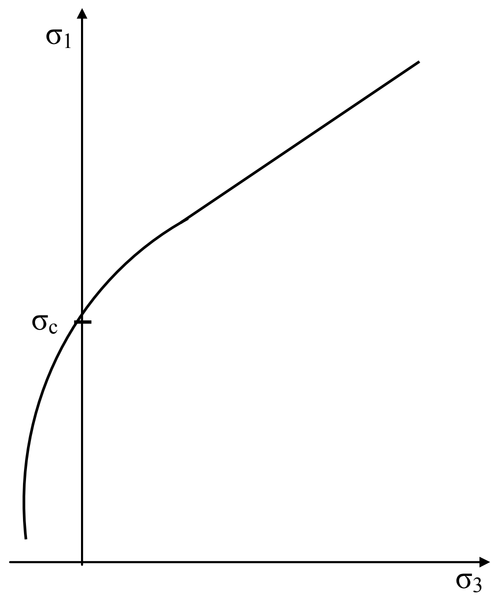


Figure 2: Non-linear Hoek-Brown failure criterion

Recently, a novel method [4] was proposed to obtain the equivalent angle of dilation for a non-linear plastic potential function corresponding to the H-B failure criterion expressed as

$$Q = \sigma_1 - \sigma_3 - \sigma_c \left(m_{dil} \frac{\sigma_3}{\sigma_c} + s \right)^a \tag{5}$$

where m_{dil} is the H-B dilation parameter. $m_{dil} = m_b$ corresponds to an associated flow rule. However, the method proposed in [4] was restricted to the axisymmetric analysis of circular openings in rock mass subject to a hydrostatic in-situ stress.

The objective of this paper is to generalize the method for the analysis of circular and non-circular openings in rock mass subject to hydrostatic and non-hydrostatic in-situ stresses. The proposed method is validated by conducting elastoplastic and elastic-brittle-plastic (Figure 3) plane strain finite element analysis of several cases.

2 The Proposed Method

The novel method is developed by first considering the axisymmetric problem of a circular opening of radius r_0 in rock mass subject to a hydrostatic in-situ stress σ_0 (Figure 4). A uniformly distributed radial pressure p_0 is considered along the

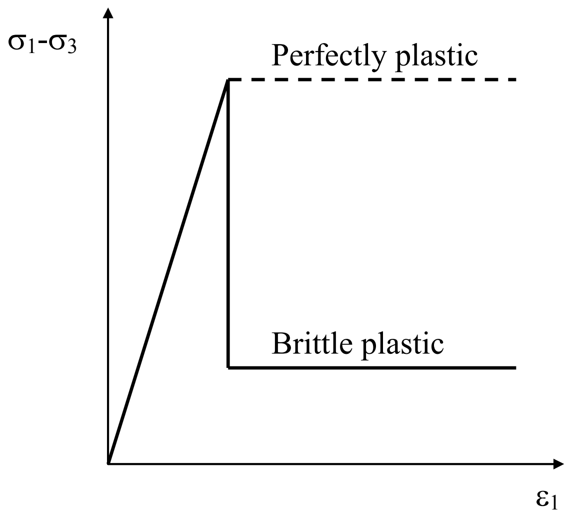


Figure 3: Stress-strain diagrams for perfectly plastic and brittle plastic behaviours

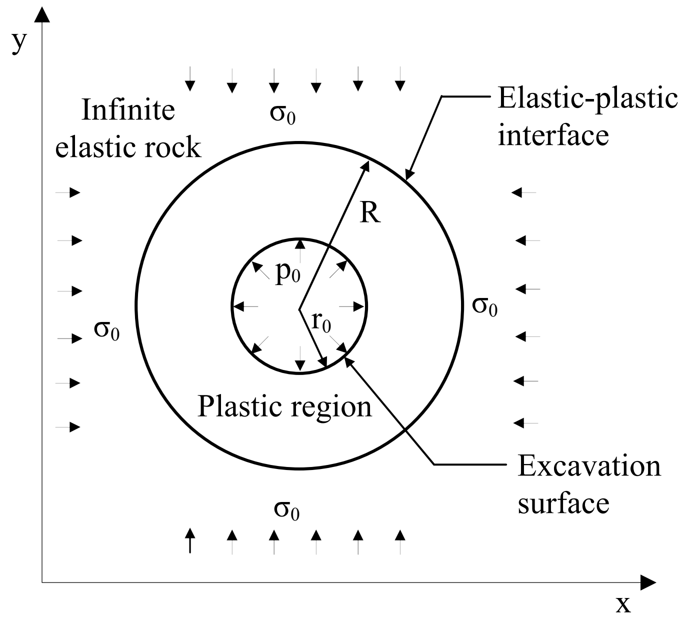


Figure 4: A circular opening in an infinite rock medium

opening surface. The equivalent angle of dilation ψ^{eq} is obtained by equating the maximum radial displacement u_{max} computed by using the M-C and H-B failure criteria as follows:

$$u_{max}^{M-C} = u_{max}^{H-B} \tag{6}$$

where the superscripts are used to denote the failure criteria used for the analysis. The maximum displacement occurs at the opening surface, r = r_0. The radial displacements u^{M-C} and u^{H-B} are obtained by solving, respectively, the following equations for M-C and H-B failure criteria:

$$\frac{du^{M-C}}{dr} + K_d^{eq}\frac{u^{M-C}}{r} = -\left(\varepsilon_r^e + K_d^{eq}\varepsilon_\theta^e\right) \tag{7}$$

$$\frac{du^{H-B}}{dr} + f\frac{u^{H-B}}{r} = -\left(\varepsilon_r^e + f\varepsilon_\theta^e\right) \tag{8a}$$

$$f = 1 + a\,m_{dil}\left(m_{dil}\frac{\sigma_r}{\sigma_c} + s\right)^{a-1} \tag{8b}$$

where r and θ are the polar coordinates, ε_r^e and ε_θ^e are the elastic components of the radial and circumferential strains given by

$$\varepsilon_r^e = \frac{1-\nu^2}{E}\left[\sigma_r - \sigma_0 - \frac{\nu}{1-\nu}\left(\sigma_\theta - \sigma_0\right)\right] \tag{9}$$

$$\varepsilon_\theta^e = \frac{1-\nu^2}{E}\left[\sigma_\theta - \sigma_0 - \frac{\nu}{1-\nu}\left(\sigma_r - \sigma_0\right)\right] \tag{10}$$

and K_d^{eq} is the EM-C dilation parameter.

A closed-form solution of Equation (7) may be found in [13]. The value of K_d^{eq} was computed by solving Equations (6) – (10) using a mathematical software. The equivalent angle of dilation was then obtained by using the following equation:

$$\psi^{eq} = sin^{-1}\left(\frac{K_d^{eq}-1}{K_d^{eq}+1}\right) \tag{11}$$

The results for K_d^{eq} are independent of the radius of the opening and therefore, the above method developed for circular openings was assumed to be applicable to non-circular openings as well.

For the analysis of underground openings in rock mass subject to non-hydrostatic in-situ stresses, it was assumed that

$$\sigma_0 = \frac{\sigma_{10} + \sigma_{30}}{2} \tag{12}$$

where σ_{10} and σ_{30} are the major and minor principal in-situ stresses, respectively.

3 Numerical Validation

In order to validate the proposed method, several example cases were tested numerically by using the finite element software Phase^2 [14]. Five different cases were considered by varying the mechanical properties of rock mass, radius of the opening and support pressures. The data were taken from published work [15, 16, 17, 18] and are shown in Table 1. The subscript r used for H-B strength parameters correspond to the residual values.

Case	r_0 (m)	E (GPa)	ν	σ_c (MPa)	m	s	m_r	s_r	p_0 (MPa)
1	5	5.5	0.25	30	1.7	0.0039	1	0	5
2	6.1	27.6	0.2	69	1.5	0.004	0.3	0	2.5
3	10	60	0.2	210	10.84	0.296	1	0.01	0
4	10	90	0.2	200	16	0.33	1	0.01	0
5	4	40	0.2	300	7.5	0.1	0.3	0.001	0

Table 1: Cases used for numerical validation

The non-hydrostatic in-situ principal stresses σ_{10} and σ_{30} used for the analysis are shown in Table 2. These correspond to σ_{10}/σ_{30} =1.5. The value of $a = a_r = 0.5$ was used and three different values of m_{dil} = 0, $m_r/8$ and $m_r/4$ were considered. Eight-noded quadrilateral elements were used and the unbounded extent of rock mass was simulated by using infinite elements.

Case	σ_{10} (MPa)	σ_{30} (MPa)	σ_0 (MPa)
1	45	30	37.5
2	31.1	20.7	25.9
3	150	100	125
4	135	90	112.5
5	162	108	135

Table 2: Stress fields used for numerical validation

For all cases, the EM-C strength parameters c and ϕ were obtained by using the BFe (best fitting in the existing stress range) procedure proposed in [9] and the equivalent angle of dilation ψ^{eq} was computed by using Equations (6) – (11).

3.1 Circular Openings

Figure 5 shows a typical finite element model for a circular opening. By using symmetry, only one quarter of the domain was analyzed. Typical comparisons of results for maximum displacements δ_{max}, distributions of displacements and yielded zones obtained by using the H-B and EM-C parameters are shown in Table 3 and Figures 6 and 7, respectively. For brevity, results for only the elastic-brittle-plastic analysis are shown herein. The yielded zones and distributions of displacements are

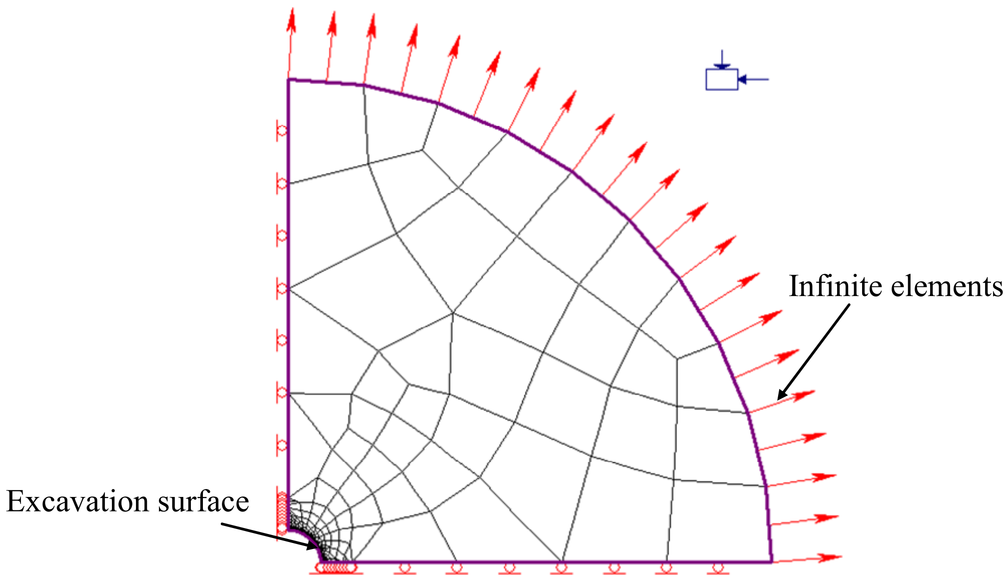
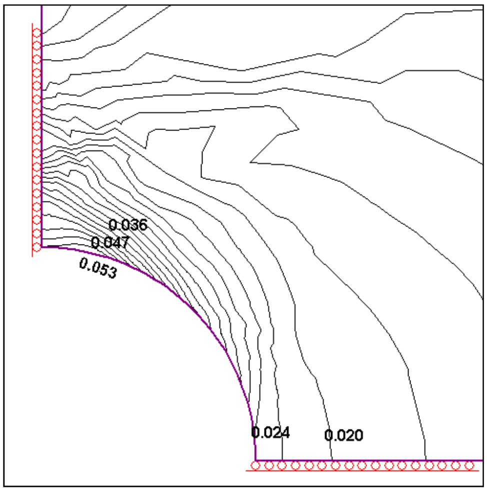


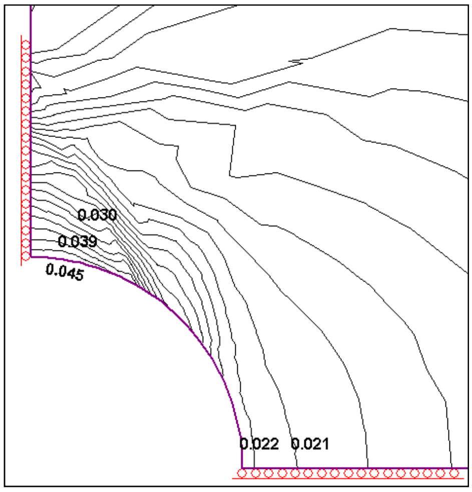
Figure 5: A typical finite element model used for a circular opening

Case	m_{dil}	ψ (°)	δmax (m)		
			H-B criterion	EM-C criterion	Difference (%)
1	0	0	0.24	0.23	4
	$m_r/8$	2.6	0.28	0.25	11
	$m_r/4$	5	0.32	0.28	13
2	0	0	0.038	0.038	0
	$m_r/8$	3	0.045	0.043	4
	$m_r/4$	5.7	0.053	0.045	15
3	0	0	0.09	0.09	0
	$m_r/8$	7.6	0.09	0.1	-11
	$m_r/4$	12.9	0.12	0.13	-8
4	0	0	0.05	0.05	0
	$m_r/8$	8.3	0.06	0.06	0
	$m_r/4$	14	0.07	0.06	14
5	0	0	0.11	0.11	0
	$m_r/8$	5.7	0.15	0.14	7
	$m_r/4$	10.1	0.19	0.17	11

Table 3: Comparison of results for maximum displacements of circular openings

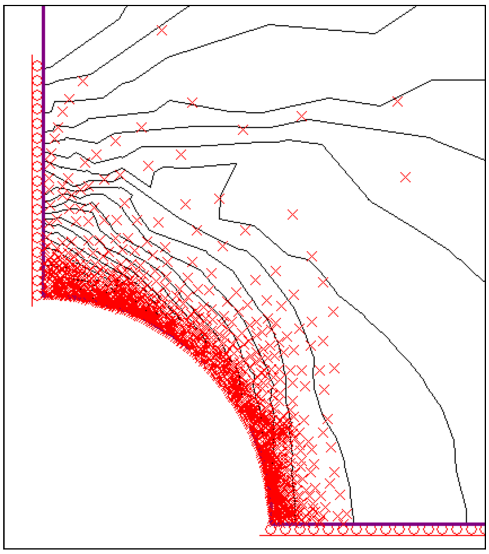


(a)

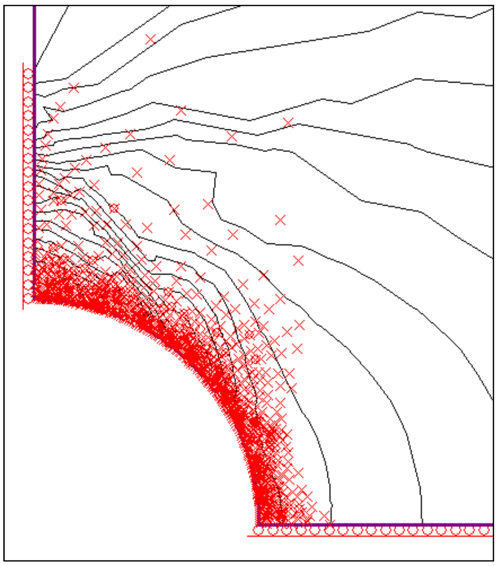


(b)

Figure 6: Distribution of displacements (m) around the circular opening in rock mass satisfying (a) H-B criterion (b) EM-C criterion (Case 2, elastic-brittle-plastic, $m_{dil} = m_r/4$, $\sigma_{10}/\sigma_{30} = 1.5$)



(a)



(b)

Figure 7: Yielded elements around the circular opening in rock mass satisfying (a) H-B criterion (b) EM-C criterion (Case 2, elastic-brittle-plastic, $m_{dil} = m_r/4$, $\sigma_{10}/\sigma_{30} = 1.5$)

presented for Case 2 with $m_{dil} = m_r/4$ only. The agreement in results was found to be very good with the maximum error in δ_{max} being 15%.

3.2 Non-Circular Opening

A D-shaped opening was considered to examine the applicability of the proposed method for the analysis of non-circular openings. Figures 8 and 9 show the dimensions of the opening and the finite element model, respectively. By using symmetry, only one half of the domain was considered. The first two cases were analyzed and the results for δ_{max} are presented in Table 4. The maximum difference in results for δ_{max} was found to be 16%. Figures 10 and 11 show, respectively, typical comparisons of results for the distribution of displacements and the yielded

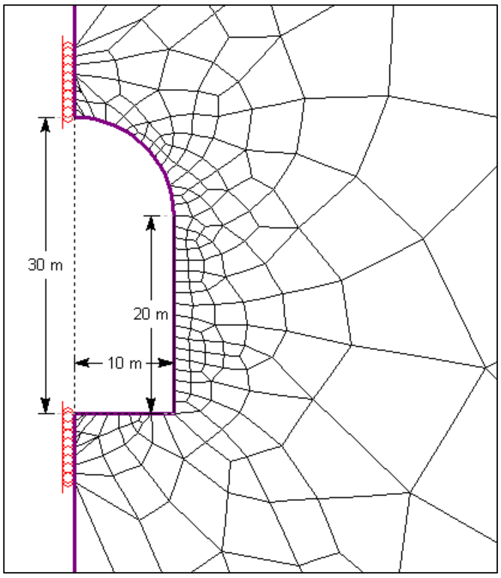


Figure 8: Dimensions of the D-shaped opening

Case	m_{dil}	K_d	ψ (°)	δ_{max} (m)		Difference (%)
				H-B criterion	EM-C criterion	
1	0	1.000	0	0.70	0.64	9
	$m_r/8$	1.096	2.6	0.81	0.70	13
	$m_r/4$	1.190	5.0	0.95	0.79	16
2	0	1.000	0	0.10	0.09	7
	$m_r/8$	1.112	3.0	0.12	0.11	9
	$m_r/4$	1.220	5.7	0.14	0.12	12

Table 4: Comparison of results for the maximum displacements of the D-shaped opening

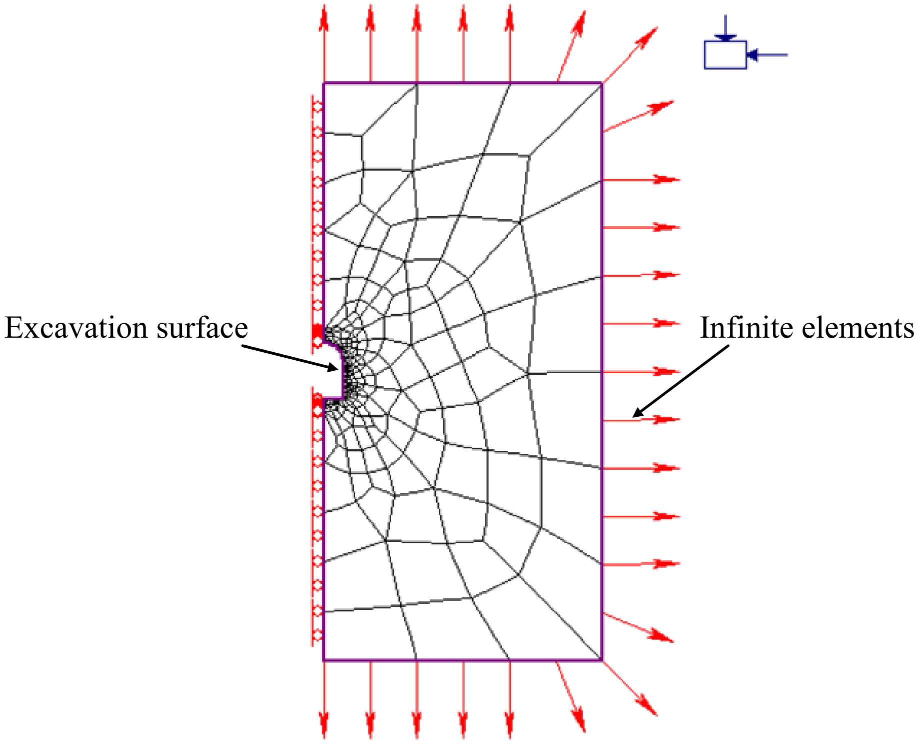


Figure 9: Finite element mesh for the D-shaped opening

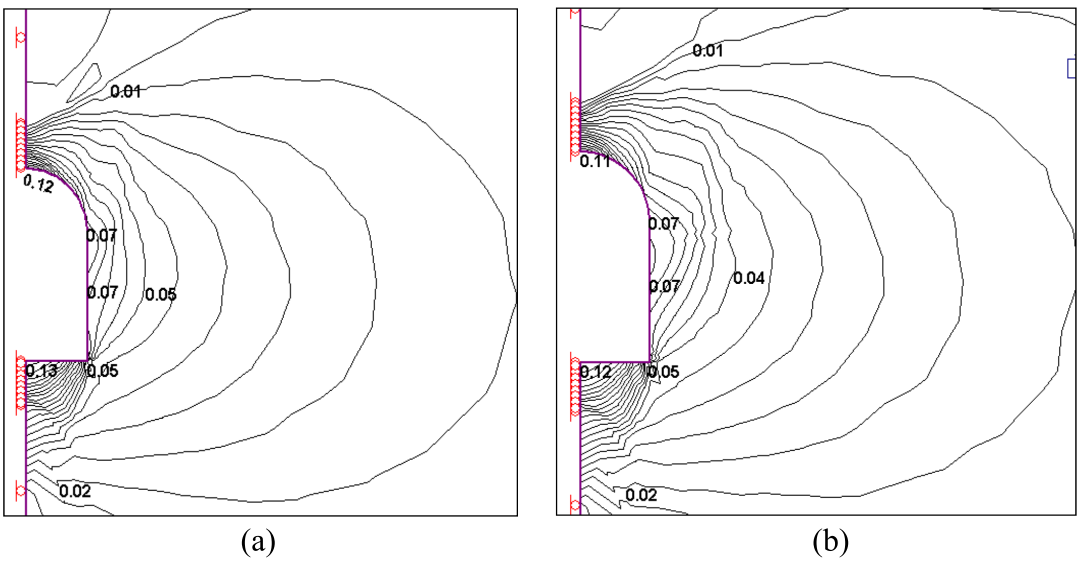
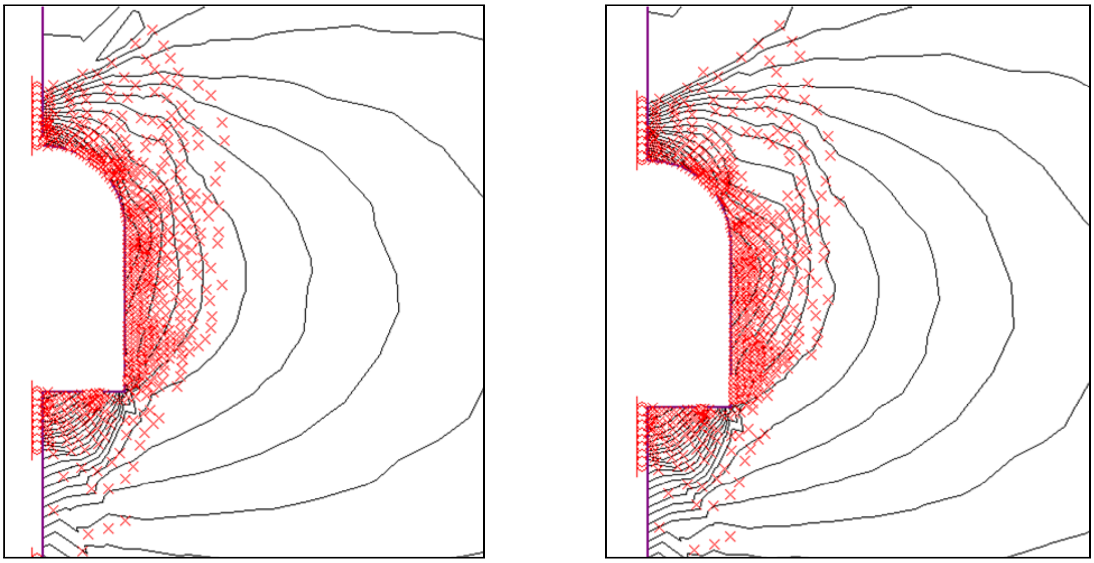


Figure 10: Distribution of displacements (m) around the D-shaped opening in rock mass satisfying (a) H-B criterion (b) EM-C criterion (Case 2, elastic-brittle-plastic, $m_{dil} = m_r/4$, $\sigma_{10}/\sigma_{30} = 1.5$)

zones for the elastic-brittle-plastic analysis of Case 2 with $m_{dil} = m_r/4$. The agreement in results was found to be very good.



(a) (b)

Figure 11: Extent of yielded zone around the D-shaped opening in rock mass satisfying (a) H-B criterion (b) EM-C criterion. (Case 8, elastic-brittle-plastic, $m_{dil} = m_r/4$, $\sigma_{10}/\sigma_{30} = 1.5$)

4 Conclusions

A novel method was proposed to compute the equivalent M-C dilation parameter for the analysis of underground openings in rock mass governed by the non-linear H-B failure criterion and subject to a non-hydrostatic in-situ stress field. The method was found to be very effective and efficient for the plane strain elastoplastic and elastic-brittle-plastic finite element analysis of circular and non-circular underground openings.

Acknowledgement

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