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Numerical Behaviour of Support Splitting and Merging in Nonlinear Diffusion Equations

K. Tomoeda
Department of Applied Mathematics and Informatics
Osaka Institute of Technology, Japan

Abstract

Numerical experiments suggest interesting properties in the several fields of fluid dynamics, plasma physics and population dynamics. Among such properties, there is a striking manifestation of *support splitting and merging phenomena* in the behaviour of non-stationary seepage. The model equation in one dimensional space is written in the form of the initial-boundary value problem with the effect of a non-linear filtration. In this paper, such phenomena are realized by use of finite difference schemes, and are justified from numerical and analytical points of view. Moreover, several interesting numerical examples are demonstrated.

Keywords: nonlinear diffusion, free boundary, interface, support splitting, support merging, difference scheme.

1 Introduction

We are concerned with the dynamical behaviour of non-stationary seepage in the nonlinear filtration. The representative filtration is well known as the flow through porous media where the water evaporates. In particular, it is expected that such a seepage exhibits *support splitting and merging phenomena*, which are caused by the interaction between the nonlinear diffusion and the penetration of the fluid from the boundary on which the flowing tide and the ebbing tide occur. Here the support means the region where the fluid exists.

To model such phenomena in one dimensional space we introduce a model based on the following equation, which is used to describe the flow through porous media with absorption [1, 2]:

$$\begin{cases} v_t(t,x) = (v^m)_{xx} - cv^p & \text{in } (0,\infty)\times(-L,\ L), \\ v(t,\pm L) = \psi_\pm(t) & \text{in } (0,\infty), \\ v(0,x) = v^0(x) & \text{in } (-L,\ L), \end{cases} \tag{1}$$

where v denotes the density of the fluid, $m > 1$, $0 < p < 1$, $c > 0$, $m + p = 2$ and $v^0(x), \psi_\pm(t) \geq 0$. This equation is also used to describe the propagation of thermal waves in plasma physics [3].

From analytical points of view, the existence and uniqueness of a weak solution and the comparison theorem are proved by Oleinik, Kalashnikov and Chzou [4], Kalashnikov [5, 6] and Knerr [7] in the case of the initial value problem, and by Bertsch [8] in the case of the initial-boundary value problem.

For the initial value problem, Rosenau and Kamin [3] suggested *support splitting phenomena* in several numerical examples. This motivates us to develop an interface tracking algorithm. By using our scheme [9] based on this algorithm we found *support splitting and merging phenomena*. We also constructed the initial function for which *repeated support splitting and merging phenomena* appear [10]. For the initial-boundary value problem (1) Kersner proved the appearance of *support splitting phenomena* [11], but he did not show that *support merging phenomena* appear after the support splits.

To investigate such phenomena in the problem (1) it is important to construct a numerical method to (1) and to analyze the profile of the support of the stationary solution $w(x)$ satisfying

$$\begin{cases} (w^m)_{xx} - cw^p = 0 & \text{in } (-L,\ L), \\ w(-L) = \alpha', w(L) = \beta', \end{cases} \tag{2}$$

where α' and β' are positive constants. Moreover, we prove the stabilization of the solution $v(t,x)$ of (1); that is, $v(t,x)$ converges to the stationary solution $w(x)$ as $t \to \infty$.

2 Finite difference scheme

We put $u = v^{m-1}$ and rewrite (1) as follows:

$$\begin{cases} u_t = muu_{xx} + \dfrac{m}{m-1}(u_x)^2 - (m-1)c\chi_{u>0} & \text{in } (0,\infty)\times(-L,\ L), \\ u(t,\pm L) = (\psi_\pm(t))^{m-1} & \text{in } (0,\infty), \\ u(0,x) = u^0(x) \equiv (v^0(x))^{m-1} & \text{in } (-L,\ L), \end{cases} \tag{3}$$

where the term of absorption is written as the constant $-(m-1)c\chi_{u>0}$ by the assumption $m + p = 2$. Our scheme approximates the problem (3) instead of (1). Let h be a space mesh width and V_h be the set of the nonnegative and piecewise-linearly interpolated functions $u_h = u_h(x)$ with the mesh $\mathcal{M}_h = \{-Nh,\ -(N-1)h, \cdots,\ (N-1)h,\ Nh\}$, where N is an integer and $h = \frac{L}{N}$. The scheme is described as follows:

Find the sequence $\{u_h^n\}_{n=1,2,\cdots} \subset V_h$ with the mesh $\mathcal{M}_h$ for each $u_h^0 \in V_h$ such that

$$\begin{cases} u_h^{n+1} = P_{h,k} D_{h,k} H_{h,k} u_h^n & \text{for } n = 0, 1, 2, \cdots, \\ u_h^n(-L) = (\psi_-(t_n))^{m-1},\ u_h^n(L) = (\psi_+(t_n))^{m-1} & \text{for } n = 0, 1, 2, \cdots, \\ u_h^0(ih) = u^0(ih) & \text{for } i = 0, \pm 1, \cdots, \pm N, \end{cases} \tag{4}$$

where $P_{h,k}$, $D_{h,k}$ and $H_{h,k}$ are difference operators approximating $u_t = muu_{xx}$, $u_t = -(m-1)\chi_{u>0}$ and $u_t = \dfrac{m}{m-1}(u_x)^2$, respectively. Since these difference operators are written in somewhat complicated form, we omit their description [9]. The variable time step $k = k_{n+1} \equiv t_{n+1} - t_n$ $(t_0 = 0)$ is determined by

$$k = \frac{(m-1)h}{4m\|(u_h)_x^n\|_{L^\infty}} \ . \tag{5}$$

Without proof we state Theorems 1 and 2. The latter can be derived from the former.

Theorem 1(Basic estimates [9])**.** *Assume that* $u^0(x) \in C^2[-L,\ L]$ *be a nonnegative function satisfying*

$$(u^0(x))_{xx} \geq 0, \tag{6}$$

$$\|(u^0(x))_x\|_{L^\infty} \leq (m-1)\sqrt{\frac{c}{m}}, \tag{7}$$

and that

$$\psi_-(t) = \alpha' \quad \text{and} \quad \psi_+(t) = \beta', \tag{8}$$

where $\alpha' = (u^0(-L))^{\frac{1}{m-1}}$ *and* $\beta' = (u^0(L))^{\frac{1}{m-1}}$ *be arbitrary positive constants. Then*

$$0 \leq u_h^n(x) \leq \|u_h^0\|_{L^\infty}, \tag{9}$$

$$\|(u_h^n)_x\|_{L^\infty} \leq (m-1)\sqrt{\frac{c}{m}}, \tag{10}$$

$$TV((u_h^n)_x) \leq 2(m-1)\sqrt{\frac{c}{m}}, \tag{11}$$

$$\|(u_h^{n+1} - u_h^n)/k_{n+1}\|_{L^1[-L,\ L]} \leq 2m\|u_h^0\|_{L^\infty}(m-1)\sqrt{\frac{c}{m}} + 4L(m-1)c. \tag{12}$$

Theorem 2 (Convergence of numerical solutions [9])**.** *Under the assumption of Theorem 1 let* $\{h\}$ *be an arbitrary sequence which tends to zero. Then, there exists the unique weak solution* v *of (1), and*

$$\|v_h - v\|_{L^\infty(\mathcal{H})} \longrightarrow 0 \quad \text{as } h \to 0, \tag{13}$$

where $\mathcal{H} \subset [0, \infty) \times [-L,\ L]$ *is an arbitrary fixed compact set,* $v_h = (u_h)^{1/(m-1)}$, $u_h(t, x) = u_h^n(x)$ *on* $[t_n, t_{n+1}) \times [-L,\ L]$ *for all* t_n *and* h.

3 Stationary solutions

We state the results for the stationary solutions of (2).

Theorem 3. *Let* $\alpha \equiv (\alpha')^{m-1}$ *and* $\beta \equiv (\beta')^{m-1}$ *be arbitrary positive constants satisfying*

$$\beta \leq \alpha + (m-1)\sqrt{\frac{c}{m}}\,(2L), \tag{14}$$

$$\max\left\{\alpha - (m-1)\sqrt{\frac{c}{m}}(2L), 0\right\} \leq \beta. \tag{15}$$

Then there exists a unique stationary solution $w(x) \geq 0$ *of (2).*

Moreover, we have

Theorem 4. *Let* $\alpha \equiv (\alpha')^{m-1}$ *and* $\beta \equiv (\beta')^{m-1}$ *be arbitrary positive constants such that*

$$\beta \leq \alpha + (m-1)\sqrt{\frac{c}{m}}\,(2L). \tag{16}$$

1) If

$$0 \leq \alpha - (m-1)\sqrt{\frac{c}{m}}\,(2L) \leq \beta, \tag{17}$$

then $w(x) > 0$ *on* $[-L,\ L]$*, which implies that the support never splits.*

2) If there exists $\ell(-L < \ell < L)$ *satisfying*

$$\alpha - (m-1)\sqrt{\frac{c}{m}}\,(\ell + L) = 0, \qquad (m-1)\sqrt{\frac{c}{m}}\,(L - \ell) < \beta, \tag{18}$$

then $w(x) > 0$ *on* $[-L,\ L]$*, which implies that the support never splits.*

3) If there exists $\ell(-L < \ell < L)$ *satisfying*

$$\alpha - (m-1)\sqrt{\frac{c}{m}}\,(\ell + L) = 0, \qquad \beta < (m-1)\sqrt{\frac{c}{m}}\,(L - \ell), \tag{19}$$

then $w(x) = 0$ *on* $[\ell,\ \ell^*]$ *for some* $\ell^*(\ell < \ell^* < L)$*, which implies the support splitting phenomena.*

Proofs of Theorems 3 and 4.

To prove the existence and the uniqueness of the solution $w(x)$ of (2), we rewrite the original equation as follows:

$$\begin{cases} \dot{\phi}(x) = z(x) \\ \dot{z}(x) = \dfrac{(m-1)c - a(z(x))^2}{m\phi(x)}, \end{cases} \tag{20}$$

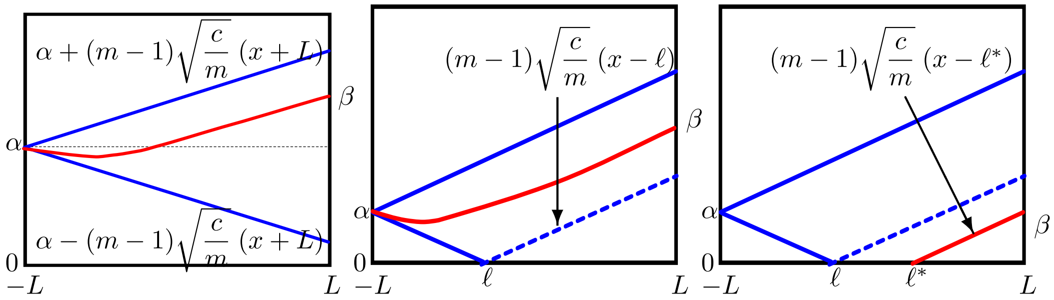

Figure 1: Stationary solutions $w^{m-1}(x)$ in Cases 1), 2) and 3).

where $\phi(x) = (w(x))^{m-1}$, $z(x) = ((w(x))^{m-1})_x$, $a = \frac{m}{m-1}$, and the boundary conditions are given by

$$\phi(-L) = \alpha \quad \text{and} \quad \phi(L) = \beta. \tag{21}$$

Without loss of generality we may take $m = 1.5$, $p = 0.5$ and $c = 6$. Then we have $(m-1)\sqrt{\frac{c}{m}} = 1$.

We first consider the existence of the global solutions $\phi(x)$ and $z(x)$ $(x > -L)$ of the initial value problem (20) with

$$\phi(-L) = \alpha \ (> 0) \quad \text{and} \quad z(-L) = \gamma \ (-1 < \gamma < 1). \tag{22}$$

It is obvious from the standard theory of ordinary differential equations that the local solution of (20) always exists and is unique at an arbitrary point $x = \xi \geq -L$ if $\phi(\xi) > 0$. We prove that

$$\phi(x) > 0 \quad \text{and} \quad |z(x)| < 1 \quad \text{for } x \geq -L. \tag{23}$$

Assume the contrary; that is, suppose there exists $\theta^*(> -L)$ satisfying the following each case.

Case i) $\phi(\theta^* - 0) = 0$ and $\phi(x) > 0$ for $x < \theta^*$;

Case ii) $|z(\theta^* - 0)| = 1$ and $|z(x)| < 1$ for $x < \theta^*$;

Case iii) The solution fails to exist at $x = \theta^*(> -L)$;

We consider Case i). Since $|z(x)| < 1 (x < \theta^*)$, it follows from (20) that

$$-1 < z(x) = \frac{-1 + e^{\int_{-L}^{x} \frac{4}{\phi(\eta)} d\eta + E}}{1 + e^{\int_{-L}^{x} \frac{4}{\phi(\eta)} d\eta + E}} < 1 \quad \text{on } [-L,\ \theta^*), \tag{24}$$

where $E = \log \frac{1+\gamma}{1-\gamma}$. From (20) we have $\dot{z}(x) = \ddot{\phi}(x) > 0$. Thus $\phi(x)$ becomes convex downward within $[-L,\ \theta^*)$, and

$$e^{\int_{-L}^{x} \frac{4}{\phi(\theta)} d\theta + E} > e^{\int_{-L}^{x} \frac{4}{\theta^* - \theta} d\theta + E} \to \infty \quad \text{as } x \nearrow \theta^*. \tag{25}$$

Hence $z(x) \nearrow 1$ as $x \nearrow \theta^*$, which contradicts the choice of θ^*.

In Case ii) the inequality (24) also holds, because $\phi(x) > 0 (x < \theta^*)$. Since $\phi(\theta^* - 0) > 0$, it follows that $z(\theta^* - 0)$ exists and $z(\theta^* - 0) < 1$, which yields a contradiction.

In Case iii) the inequality (24) also holds and gives

$$\max\{\alpha - (x+L), 0\} < \phi(x) < \alpha + (x+L) \quad \text{on } [-L,\ \theta^*). \tag{26}$$

Thus $\phi(x)$ becomes convex downward within $[-L,\ \theta^*)$, and $\phi(\theta^* - 0)$ and $z(\theta^* - 0)$ exist. Hence $\phi(x)$ and $z(x)$ can be continued to the right of θ^*, which contradicts the choice of θ^*. From the above consideration the inequality (23) follows.

Next we show the existence of the solution of (20)-(21). In the case where (17) or (18) holds we apply the continuous dependence of the solution on the initial value to the proof. In (22) we fix α and change γ. Let $\phi_{\alpha,\gamma}(x)$ and $z_{\alpha,\gamma}(x)$ denote the solutions of (20) with the initial values $\phi_{\alpha,\gamma}(-L) = \alpha$ and $z_{\alpha,\gamma}(-L) = \gamma$. We note that $\phi_\alpha(x) = \alpha - (x+L)$ $(x > -L)$ and $\phi_\beta(x) = \beta - (L-x)$ $(x < L)$ become the solutions of (20). These lines intersect at some point $x = x^*$ and $\phi_\alpha(x^*) = \phi_\beta(x^*) > 0$. Take $\gamma_1 (> -1)$ sufficiently close to -1. The solution $\phi_{\alpha,\gamma_1}(x)$ is positive for all $x > -L$, and $\phi_{\alpha,\gamma_1}(x) < \phi_\beta(x)$ for $x > {}^\exists\tilde{x}$ $(x^* < \tilde{x} < L)$. Take $\gamma_2 (< 1)$ sufficiently close to 1. The solution $\phi_{\alpha,\gamma_2}(x)$ is positive and $\phi_{\alpha,\gamma_2}(x) > \phi_\beta(x)$ for $x > -L$. Thus $\phi_{\alpha,\gamma_1}(L) < \beta < \phi_{\alpha,\gamma_2}(L)$. By the continuous dependence of $\phi_{\alpha,\gamma}(L)$ on γ there exists some number γ^* such that the positive solution $\phi_{\alpha,\gamma^*}(x)$ exists and connects two points $(-L, \alpha)$ and (L, β). Moreover, $w(x) = (\phi_{\alpha,\gamma^*}(x))^{\frac{1}{m-1}}$ becomes the solution of (2).

In the case where (19) holds we can construct the solution:

$$\phi(x) = \begin{cases} \alpha - (x+L) & \text{on } [-L,\ \ell], \\ 0 & \text{on } [\ell,\ \ell^*], \\ \beta - (L-x) & \text{on } [\ell^*,\ L], \end{cases} \tag{27}$$

where $\ell = -L + \alpha$ and $\ell^* = L - \beta$.

Finally we show the uniqueness of the solution $\phi(x)$ in the following sense:
If two solutions $\phi(x)$ and $\psi(x)$ satisfy $\phi(-L) = \psi(-L) = \alpha$ and $\phi(L) = \psi(-L) = \beta$, then $\phi(x) = \psi(x)$ on $[-L, L]$.
We prove it. Suppose that there exists ξ $(-L < \xi \le L)$ satisfying $\phi(\xi) = \psi(\xi)$ and $\phi(x) > \psi(x)$ on $(-L, \xi)$. Put $w_\phi(x) = (\phi(x))^{\frac{1}{m-1}}$ and $w_\psi(x) = (\psi(x))^{\frac{1}{m-1}}$. Then these solutions satisfy (2), and we have

$$\begin{aligned} {w_\phi}^m(\xi) - {w_\psi}^m(\xi) &= {w_\phi}^m(-L) - {w_\psi}^m(-L) + \int_{-L}^{\xi} ({w_\phi}^m(\theta))_x - ({w_\psi}^m(\theta))_x d\theta \\ &= \int_{-L}^{\xi} ({w_\phi}^m(\theta))_x - ({w_\psi}^m(\theta))_x d\theta, \end{aligned} \tag{28}$$

$$\begin{aligned} ({w_\phi}^m(\theta))_x - ({w_\psi}^m(\theta))_x &= ({w_\phi}^m(-L))_x - ({w_\psi}^m(-L))_x + \int_{-L}^{\theta} ({w_\phi}^m(\eta))_{xx} - ({w_\psi}^m(\eta))_{xx} d\eta \\ &\ge \int_{-L}^{\theta} (c{w_\phi}^p(\eta) - c{w_\psi}^p(\eta)) d\eta > 0 \ \text{ for } \theta \in (-L,\ \xi]. \end{aligned} \tag{29}$$

Hence, $w_\phi{}^m(\xi) - w_\psi{}^m(\xi) > 0$, which yields $\phi(\xi) - \psi(\xi) > 0$. This is a contradiction. Thus the proof is complete.

Theorem 5(Stabilization). *Under the same assumptions as stated in Theorem 3, the solution $v(t,\cdot)$ of (1) with $\psi_-(t) = \alpha'$ and $\psi_+(t) = \beta'$ converges to the unique stationary solution $\tilde{w}(x)$ of (2) in $C[-L', L']$ as $t \to \infty$, where $[-L',\ L'] \subset (-L,\ L)$ is an arbitrary fixed interval.*

Proof. For the solution $v(t,\cdot)$ we consider a continuous orbit $\gamma = \{v(t,\cdot) : t \geq 0\}$ in $C[-L',\ L']$. Let ω be the ω-limit set of γ defined by

$$\omega = \{w(x) \in C[-L',\ L'] :\ {}^\exists\{t_n\},\ {}^\exists t_n \to \infty \text{ as } n \to \infty, \text{ such that } v(t_n,\cdot) \to w(x) \text{ in } C[-L',\ L'] \text{ as } n \to \infty\}.$$

By a result of DiBenedetto [12], γ is precompact in $C[-L',\ L']$; that is,

$${}^\exists\{t_n\},\ {}^\exists\hat{v}(x):\quad v(t_n,\cdot) \to \hat{v}(x) \in \omega \ \text{ in } C[-L',\ L'] \ \text{ as } \ n \to \infty.$$

On the other hand, the following inequality is proved for the solutions $v_1(t,\cdot)$ and $v_2(t,\cdot)$ of (1) by Bertsch [8]:

$$\|v_1(t) - v_2(t)\|_{L^1[-L,\ L]} \leq e^{Kt}\|v_1(0) - v_2(0)\|_{L^1[-L,\ L]} \text{ for } t \geq 0, \tag{30}$$

where K is the constant number satisfying

$$(-s^p) - (-r^p) \leq K(s - r) \text{ for any } (0 \leq r \leq s). \tag{31}$$

In general, $K = 0$. However, taking the boundedness of these solutions into consideration, we can take $|K| << 1 (K < 0)$, and for $t \geq 0$

$$\|v(t_n) - \tilde{w}\|_{L^1[-L',\ L']} \leq \|v(t_n) - \tilde{w}\|_{L^1[-L,\ L]} \leq e^{Kt_n}\|v(0) - \tilde{w}\|_{L^1[-L,\ L]}, \tag{32}$$

which tends to 0 as $n \to \infty$. Thus $\hat{v}(x) = \tilde{w}(x)\ [-L',\ L']$, and the theorem follows from the uniqueness of the stationary solution $\tilde{w}(x)$.

4 Numerical examples

We show numerical examples for (1), where $m = 1.5$, $p = 0.5$, $c = 6$ and $L = 1.5$. In figures the curves mean numerical solutions u_h which are given by the scheme (4) with $h = \frac{1}{512}$. First we try numerical computation in the case where the boundary conditions $\psi_\pm(t)$ are independent of t, and obtain three examples.

Example 1. $u(t, \pm L) = 2$, and $u(0, x) = 2$ on $[-L,\ L]$.

Example 2. $u(t, \pm L) = 1.5$, and $u(0, x) = 1.5$ on $[-L,\ L]$.

Example 3. $u(t, \pm L) = 1$, and $u(0, x) = 1$ on $[-L,\ L]$.

In each examples we find that numerical solutions converge to the stationary solutions as $t \to \infty$. Thus the properties of the dynamical behaviour of the solution stated in Theorems 3–5 are realized (see Figures 2, 3 and 4).

Next, putting $\varphi(t) \equiv \psi_\pm(t)^{m-1}$, we impose a period on $\varphi(t)$. Then some interesting phenomena are obtained.

Example 4. $u(t, \pm L) = \varphi(t) \equiv 1.5 + 0.5\cos(2\pi t)$ and $u(0, x) = 2$ on $[-L,\ L]$.
Example 5. $u(t, \pm L) = \varphi(t) \equiv 1.5 + 0.5\cos(12\pi t)$ and $u(0, x) = 2$ on $[-L,\ L]$.
Example 6. $u(t, \pm L) = \varphi(t) \equiv 1.375 + 0.375\cos(32\pi t)$ and $u(0, x) = 1.75$ on $[-L,\ L]$.
In Examples 4 *numerically repeated support splitting and merging phenomena* are observed (see Figure 5). The boundary value $\varphi(t)$ with the period 1 takes the maximum 2.0 and the minimum 1.0. On the other hand, the period is $\frac{1}{6}$ in Example 5 and is less than that in Example 4. In this example the support splitting phenomena are not observed (see Figure 6). In Example 6 $\varphi(t)$ with the period $\frac{1}{16}$ takes the maximum 1.75 and the minimum 1.0. The support begins to split at some time and never merges for all later times (see Figure 7). The numerical computation suggests that the appearance of the support splitting and merging phenomena depends on the period and the amplitude of $\varphi(t)$. So, the mathematical analysis for such phenomena is needed.

5 Conclusion

In this paper we obtain the following results in the specific case where $m > 1, 0 < p < 1$, and $m + p = 2$.
1) The convergence of numerical solutions given by our difference scheme (4) (Theorem 2);
2) The existence and uniqueness of a stationary solution of (2) (Theorems 3 and 4);
3) The stabilization of the solution of (1) with $\psi_{\pm} =$ const. (Theorem 5).

Unfortunately, in the case when $m + p \neq 2$, $m > 1$, and $0 < p < 1$, we have not obtained these results. We have not been successful in constructing the difference scheme $D_{h,k}$ satisfying the basic estimates (Theorem 1) and in proving the existence and uniqueness of the stationary solution. However, it follows that the stabilization of the solution also holds, if the stationary solution exists and is unique.

Concerning *repeated support splitting and merging phenomena* in Example 4, we may explain the mathematical justification of such a behaviour. Thus, taking the period $\frac{1}{f}$ of $u(t, \pm L) = \varphi(t) \equiv 1.5 + 0.5\cos(2\pi f t)$ sufficiently large, we can show the appearance of *repeated support splitting and merging phenomena* by Theorem 5(Stabilization). However, at this present we have no mathematical proof to justify the behaviour of the support in Examples 5 and 6.

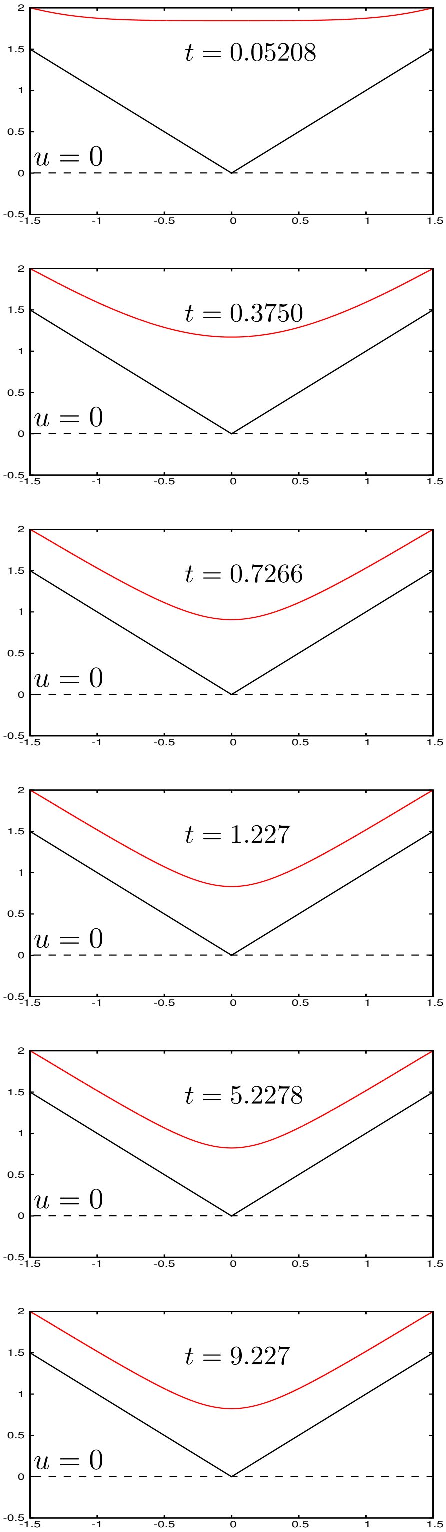

Figure 2: The convergence of numerical solutions to the stationary solution in Example 1, where $m = 1.5$, $p = 0.5$, $c = 6$ and $h = \frac{1}{512}$.

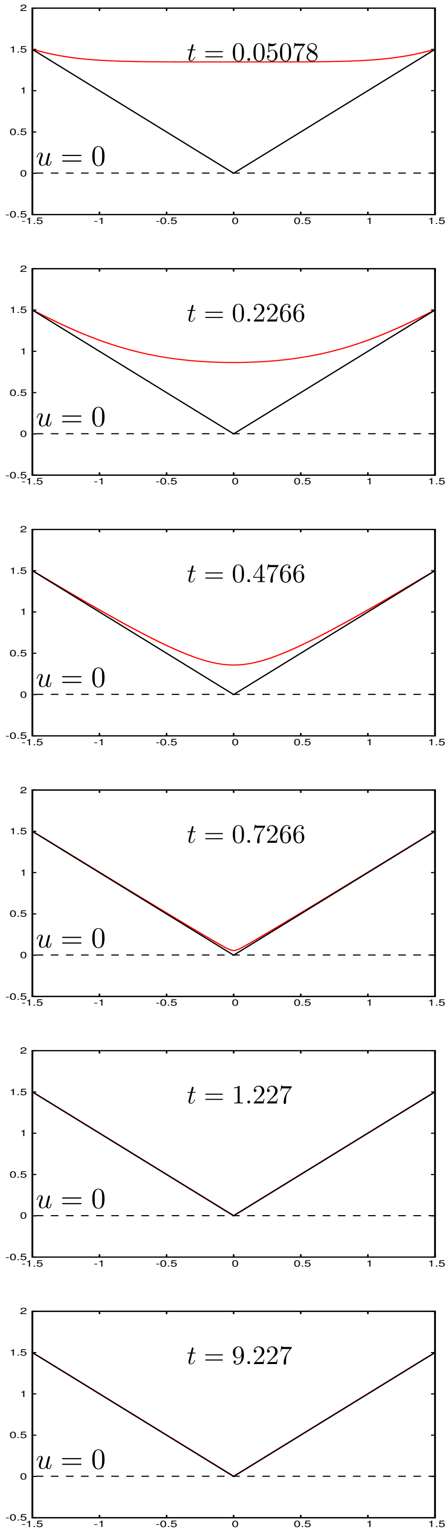

Figure 3: The convergence of numerical solutions to the stationary solution in Example 2, where $m = 1.5$, $p = 0.5$, $c = 6$ and $h = \frac{1}{512}$.

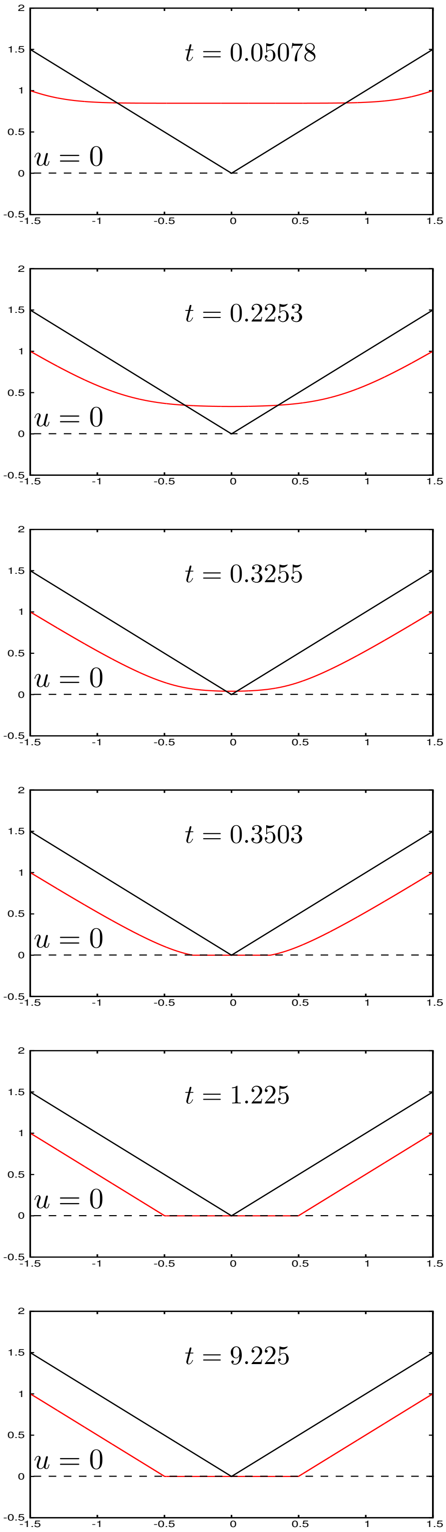

Figure 4: The convergence of numerical solutions to the stationary solution in Example 3, where $m = 1.5$, $p = 0.5$, $c = 6$ and $h = \frac{1}{512}$.

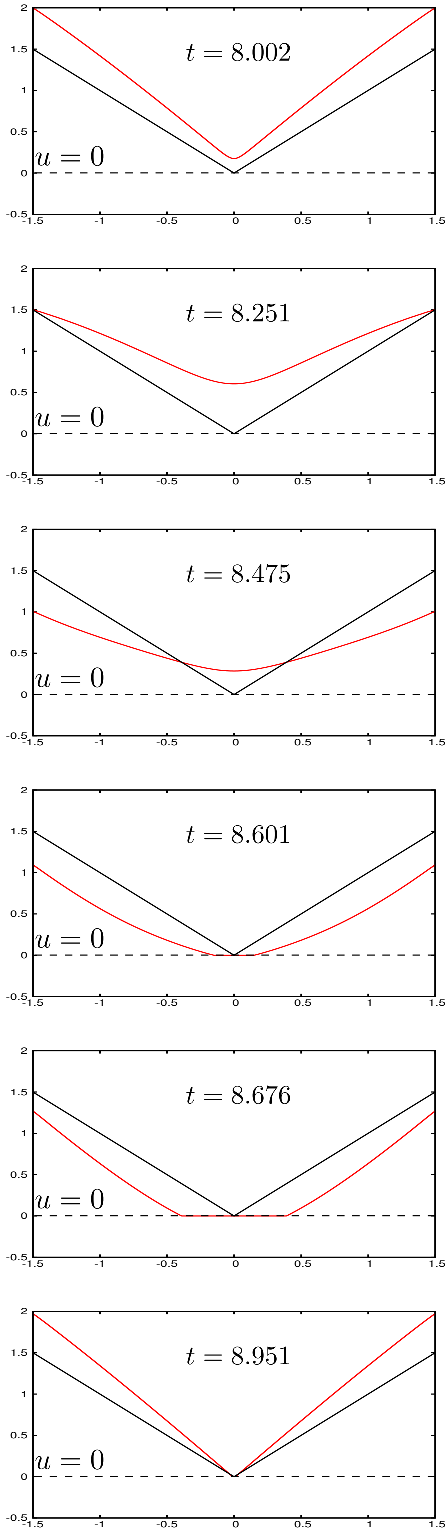

Figure 5: Numerically repeated support splitting and merging phenomena in Example 4, where $m = 1.5$, $p = 0.5$, $c = 6$ and $h = \frac{1}{512}$.

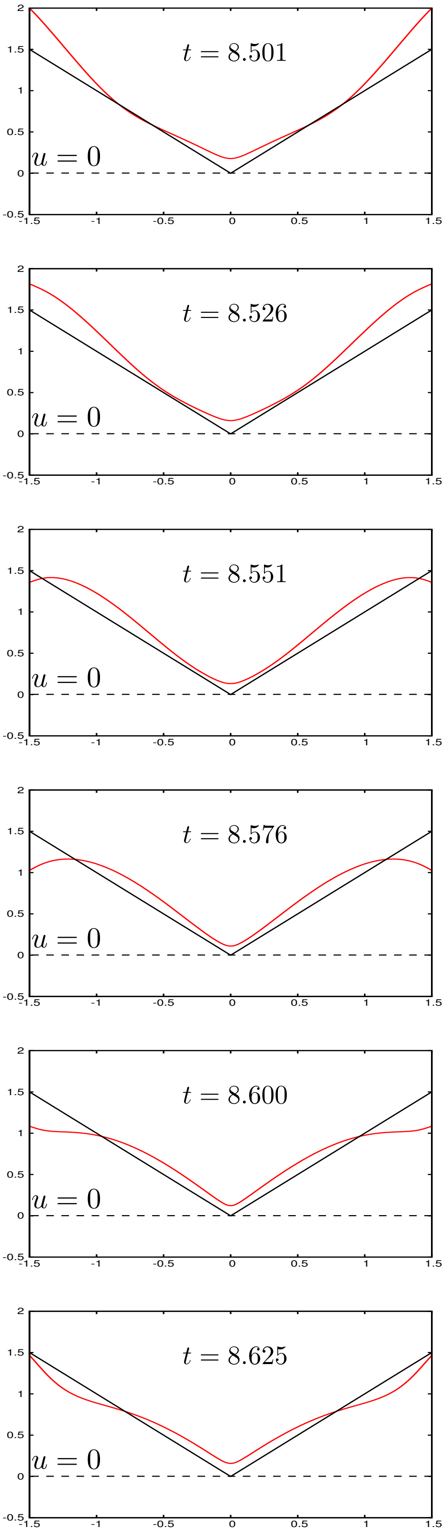

Figure 6: Numerical support no-splitting phenomena in Example 5, where $m = 1.5$, $p = 0.5$, $c = 6$ and $h = \frac{1}{512}$.

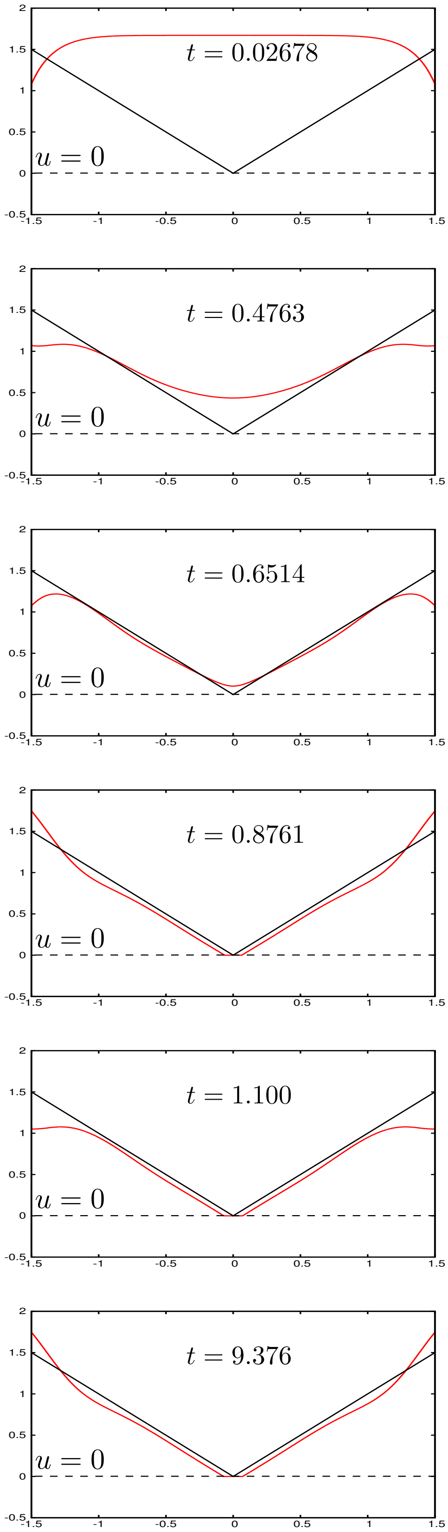

Figure 7: Numerical support no-merging phenomena in Example 6, where $m = 1.5$, $p = 0.5$, $c = 6$ and $h = \frac{1}{512}$.

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