Paper 56



©Civil-Comp Press, 2012 Proceedings of the Eighth International Conference on Engineering Computational Technology, B.H.V. Topping, (Editor), Civil-Comp Press, Stirlingshire, Scotland

Calibration of the Sensor Direction in the Simplified Three-Dimensional Digital Image Correlation Method

S.-H. Tung¹, M.-H. Shih² and W.-P. Sung³
¹Department of Civil and Environmental Engineering National University of Kaohsiung, Taiwan
²Department of Civil Engineering National Chi Nan University, Nantou, Taiwan
³Department of Landscape Architecture National Chin-Yi University of Technology, Taichung, Taiwan

Abstract

In the simplified three-dimensional digital image correlation (DIC) method, the observed object or the image capture device would be moved laterally so that two images can be taken from two different positions. The most important advantage of the simplified three-dimensional DIC method is that only one image capture device is used in this method. Therefore, the optical and mechanical properties of the image capture device will stay unchanged when the images are captured. In addition, this method is carried out under the assumption that the sensor plane of the image capture device and the movement direction are parallel. While the traditional three-dimensional DIC method uses a complicated algorithm to calculate the three-dimensional DIC method is much simpler.

If the sensor plane of the image capture device cannot be adjusted to parallel to the movement direction in an experiment, unnecessary errors could be induced. Therefore, how to make sure that the sensor plane of the image capture device is parallel to the movement direction is relatively important. To achieve this purpose, a method is proposed in this research to calibrate the sensor direction so that it can be kept parallel to the movement direction.

Keywords: simplified three-dimensional digital image correlation, direction calibration, sensor.

1 Introduction

Traditional three-dimensional DIC method [1-3] recreates the three-dimensional coordinate system of an object through two image capture devices. In addition to its complex computation, there could be also differences of mechanical and optical properties existed between these two image capture devices. This will lead to a

larger error while recreate the three-dimensional coordinates. To solve this problem, Tung et al [4] have developed a simplified three-dimensional DIC method using only one camera. The needed two images for recreate three-dimensional coordinates are captured through lateral movement of image capture device or measured object. In this simplified three-dimensional DIC method, an assumption that the sensor plane of image capture device is parallel to the direction of lateral movement is made during establishing the theory. If the assumption cannot be satisfied in the practical application, additional errors in the analysis will be induced. In order to reduce the error caused by this factor, this research utilizes two-dimensional DIC method [5-6] to calibrate the direction of the image capture device.

In this study, a digital camera is fixed upon a precise rotating table which is laid on a precise XY table. The direction of digital camera can be adjusted through the rotation of the rotating table, and two images are captured before and after the movement of camera by means of the XY table. Then the images are analyzed in order to find out the direction so that the sensor plane of the camera is parallel to the movement direction. The result shows that the measurement error can really be reduced by means of the proposed method.

2 A Simplified Three-Dimensional DIC method

Simplified three-dimensional DIC method uses only one image capture device, and the images used to recreate three-dimensional coordinates are obtained before and after a lateral movement of the image capture device or the observed object. The principle of this method can be briefly described as following.



Figure 1: Projection of a three-dimensional object before and after lateral movement [4]

Figure 1 shows that the lateral movement distance of the image capture device or the observed object is e. The projected movement length on the sensor of point a is $\overline{AA'}$, and the projected movement length on the sensor is $\overline{BB'}$ for point b which is farther from the lens than point a. Because the projected movement length is expressed in pixels, $\overline{AA'}$ and $\overline{BB'}$ can be rewritten as the following formula according to the image formation by a converging lens.

$$N_{AA'} = \gamma \left(\frac{q}{p_a}\right) e \tag{1}$$

Where $N_{AA'}$ is the movement length of point *a* on the image with a unit of pixel. γ , which has a unit of pixel/mm, is a length conversion constant from sensor to image. *q* and p_a are the image distance and object distance respectively. Since it is unable to determine γ and the image distance, a new parameter Λ combining these two parameters is defined as Equation (2).

$$\Lambda = \gamma q \tag{2}$$

The parameter Λ is unknown. However, it can be determined when an object distance difference between two points is known.

$$\Lambda = \frac{\Delta_i}{e} \left(\frac{N_{AA'} N_{II'}}{N_{AA'} - N_{II'}} \right)$$
(3)

According to Equation (3), as long as one object distance is known, then the parameter Λ can be figured out. However, there are usually inevitable errors existing in an experiment. We can calculate the average Λ from more known object distance differences to improve its accuracy.

3 Calibration method

3.1 Theory

Assume that the sensor plane and the lateral movement direction parallel to each other, as shown in Figure 2. According to the image formation theory, N_{ba} and N_{bc} will be the same when the distances between point a and b and between b and c are both equal to e.

Figure 3 shows the projection of a flat object on the sensor plane when the sensor plane and the lateral movement direction are not parallel. The include angle between sensor plane and lateral movement direction is the rotation angle (θ). The distances between point a and b and between point b and c are still equal to e.

However, the projected length $N_{ba'}$ and $N_{bc'}$ will be different, and the difference will be changed as the rotation angle θ varies.



Figure 2: Schematic drawing for the projection when sensor plane parallel to the lateral movement direction



Figure 3: Schematic drawing for the projection when sensor plane is not parallel to the lateral movement direction

In Figure 3, we assume that the object plane and the lateral movement direction are parallel. The image distance q remains unchanged. The object distance p is

changed to p' while the rotation angle θ is not equal to 0. p' can then be expressed as the following equation.

$$p' = \frac{p}{\cos\theta} \tag{4}$$

Observing ΔabO in Figure 3, the include angle between \overline{aO} and \overline{bO} is β . With the help of Sine rule, we can find the following relation.

$$\frac{e}{\sin\beta} = \frac{\overline{aO}}{\sin(\frac{\pi}{2} + \theta)}$$
(5)

Equation (5) can be rewritten as the following form with the help of Cosine rule.

$$\frac{e}{\sin\beta} = \frac{\sqrt{e^2 + p'^2 - 2ep'\cos\left(\frac{\pi}{2} + \theta\right)}}{\sin\left(\frac{\pi}{2} + \theta\right)}$$
(6)

Through transposition, the angle β can be express as the following equation.

$$\beta = \sin^{-1} \left(\frac{e \sin\left(\frac{\pi}{2} + \theta\right)}{\sqrt{e^2 + p'^2 - 2ep' \cos\left(\frac{\pi}{2} + \theta\right)}} \right)$$
(7)

Similarly, the angle α can be written as the following form by observing ΔbcO .

$$\alpha = \sin^{-1} \left(\frac{e \sin\left(\frac{\pi}{2} - \theta\right)}{\sqrt{e^2 + p'^2 - 2ep' \cos\left(\frac{\pi}{2} - \theta\right)}} \right)$$
(8)

Due to the rotation angle θ of the sensor plane, the principal axis has also a rotation angle θ . The projected lengths of \overline{bc} and \overline{ba} on the sensor are e_1 and e_2 respectively. Applying the Sine rule to $\Delta a'bO$, then we can fine the following relationship.

$$\frac{e_1}{\sin\beta} = \frac{p'}{\sin\left(\frac{\pi}{2} - \beta\right)}$$
(9)

Through transposition, Equation (9) can be rewritten as the following equation.

$$e_1 = p' \tan \beta \tag{10}$$

Similarly, e_2 can be expressed as

$$e_2 = p' \tan \alpha \tag{11}$$

The projected length of \overline{ab} on the image is $N_{ba'}$. It can be expressed as the following equation according to the image formation theory by a converging lens.

$$N_{ba'} = \frac{qe_1}{p'}\gamma \tag{12}$$

Similarly, $N_{bc'}$ can be expressed as the following equation.

$$N_{bc'} = \frac{qe_2}{p'}\gamma \tag{13}$$

As the rotation angle varies, $N_{ba'}$ and $N_{bc'}$ will also change. Theoretically, the sum of $N_{ba'}$ and $N_{bc'}$ has a maximum when there is no directional deviation. In addition, $N_{ba'}$ and $N_{bc'}$ will be identical when the directional deviation is zero. That is to say, $N_{ba'} - N_{bc'}$ is equal to zero at this time. Therefore, we have two options to identify the position without directional deviation. They are the positions where $N_{ba'} + N_{bc'}$ has a maximum or $N_{ba'} - N_{bc'}$ equals zero.

3.2 Verification

A group of simulation data are used to verify the above theory. Assume that the image distance q is fixed at 60 mm, the object distance p when there is no directional deviation is 480 mm, the object is moved laterally 10 mm (e). Substitute these values into Equation (12) and (13), and they yield different $N_{ba'}$ and $N_{bc'}$ as the rotation angle θ changes. The relationships of $N_{ba'} + N_{bc'}$ and $N_{ba'} - N_{bc'}$ versus rotation angle θ are displayed in Figure 4 and 5.

Figure 4 shows that there is a maximum at the position without directional deviation, and the curve can be perfect fitted with a second-order polynomial. Figure 5 shows that the relationship between $N_{ba'} - N_{bc'}$ and rotation angle is a straight line. This line passes the origin. So the theories in section 3.1 are proved. Therefore, we can use these two proposed methods to find the position where the sensor and the lateral movement direction are parallel to each other.



Figure 4: $N_{ba'} + N_{bc'}$ versus rotation angle



rotation angle(minute)

Figure 5: $N_{ba'} - N_{bc'}$ versus rotation angle

4 Experiment

4.1 Setup

A flat rectangular iron plate is taken as the specimen. The black and white structural speckles are sprayed on its surface. The specimen is shown in Figure 6. In the experiment, the following equipments are used.

- 1. Digital single-lens reflex camera: to capture images.
- 2. Precise rotating table: to control the rotation angle of the camera.
- 3. Precise XY table: to move the camera laterally.
- 4. Fixture: to fix the specimen.
- 5. Computer: to analyze the $N_{ba'}$ and $N_{bc'}$.

The complete setup is shown in Figure 7.



Figure 6: Specimen



Figure 7: Experimental setup

4.2 The experimental procedure

The experiment is carried out in the following procedure.

- 1. Specimen preparation: A flat iron plate is taken as the specimen. Structural speckle is marked on the surface of the specimen with spray paint.
- 2. Fix specimen: The specimen is fixed on a fixture. The plane of the specimen is adjusted to parallel the direction of lateral movement.

- 3. Fix camera: At first, the XY table is laid on the experimental desk. Then the rotating table is put on the XY table and the camera is fixed on top of the rotating table.
- 4. Choose an initial position of the camera: A direction is chosen for the camera by visually estimating so that the principal axis of the lens is approximately parallel to the plane of the specimen.
- 5. Capture images: The camera is rotated to the pre-defined angles, and the XY table moves laterally 10 mm to take the images before and after movement at every direction.
- 6. Analyze $N_{ba'}$ and $N_{bc'}$: The middle point on the image before movement is chosen, and the corresponding position after movement is analyzed. The distance between these two points is $N_{ba'}$. Then the middle point on the image after movement is also chosen, and its corresponding position before movement is analyzed. The distance between them is $N_{bc'}$.
- 7. Create graphs: Create a graph to display the relationship between $N_{ba'} + N_{bc'}$ and rotation angle. Find the rotation angle where $N_{ba'} + N_{bc'}$ has a maximum. Similarly, a graph to display the relationship between $N_{ba'} N_{bc'}$ and rotation angle is also created. Find the position where $N_{ba'} N_{bc'}$ equals to zero.

5 Results and discussion

As described in the above sections, there are two methods to find the position where the sensor is parallel to the lateral movement direction. The experimental results and discussion are described below.

1.
$$N_{ba'} + N_{bc'}$$

The experiment is carried out in according to the procedure described in section 4.2. The pre-defined rotation angles are 0°, $\pm 20'$, $\pm 40'$, $\pm 1^{\circ}$, $\pm 2^{\circ}$ and $\pm 3^{\circ}$. The analyzed $N_{ba'} + N_{bc'}$ values versus the corresponding rotation angles are plotted in Figure 8. A second-order polynomial is used to fit the obtained curve. It shows that the R square has only a value of 0.9553. This means the measured curve deviates from the regression curve. Observing these two curves, we can find that the maximum value of the measured data is not consistent with that of the regression curve. It shows the measurement error can seriously affect the experimental result. Hence, the result obtained with $N_{ba'} + N_{bc'}$ is unreliable.

2. $N_{ba'} - N_{bc'}$

The $N_{ba'} - N_{bc'}$ values versus the rotation angles obtained from the same experiment in the last section are plotted in Figure 9. A straight line is used to fit these points. The R square value is 0.9994. It shows that the measured curve and the regression curve are very consistent with each other. In comparison with the result

of $N_{ba'} - N_{bc'}$, the measurement error is not so sensitive in the method. The rotation angle where the $N_{ba'} - N_{bc'}$ is zero is about -16.9'. That is to say, the camera should be rotate to -16.9' so that the sensor plane is parallel to the lateral movement direction.



Figure 8: $N_{ba'} + N_{bc'}$ versus rotation angle



Figure 9: $N_{ba'} - N_{bc'}$ versus rotation angle

6 Conclusions

Simplified three-dimensional DIC is a non-contact measurement technique. Its computation is much simpler than the traditional three-dimensional DIC. However, simplified three-dimensional DIC has an assumption that the sensor plane of the

image capture device must parallel the lateral movement of the specimen or the image capture device. Therefore, how to adjust the direction of the image capture device is studied in this research. The following conclusions can be drawn according to the analysis results:

- 1. Theoretically, two methods derived in this research can be used to identify the direction where the sensor plane and the lateral movement direction are parallel.
- 2. The first method is to find the direction where $N_{ba'} + N_{bc'}$ has a maximum. However, it is serious affected by the measurement error of DIC. Consequently, this method is relative unreliable.
- 3. The second method is to find the direction where $N_{ba'} N_{bc'}$ is equal to zero. The analysis result shows that the regression curve is certainly a straight line. This is consistent with the result derived in the theory. In addition, the measurement error of DIC seems not to affect the result. Hence, this method is much reliable than the first one.

Acknowledgement

The authors would like to acknowledge the support of Taiwan National Science Council through grant No. NSC 100-2625-M-390-001.

Reference

- [1] P.F. Luo, Y.J. Chao, M.A. Sutton and W.H. Peters, "Accurate measurement of three-dimensional deformations in deformable and rigid bodies using computer vision", Experimental Mechanics, 33(2), 123-32, 1993.
- [2] Z.L. Kahn-Jetter and T.C. Chu, "Three-dimensional displacement measurements using digital image correlation and photogrammic analysis", Experimental Mechanics, 30(1), 10-6, 1990.
- [3] D. Winter, Optische Verschiebungsmessung nach dem Objektrasterprinzip mit Hilfe eines flächenorientierten Ansatzes, PhD thesis, Technische Universität Braunschweig, Germany; February 1993.
- [4] S.H. Tung and M.H. Shih, "Precision Verification of a Simplified Three-Dimensional DIC Method", Optics and Lasers in Engineering, 49, 937-945, 2011.
- [5] W.H. Peters and W.F. Ranson, "Digital Imaging Techniques in Experimental Stress Analysis", Optical Engineering, 21(3), 427-432, 1982.
- [6] S.H. Tung, J.C. Kuo and M.H. Shih, "Strain Distribution Analysis Using Digital-Image-Correlation Techniques", 18th KKCNN Symposium on Civil Engineering, Taiwan, 213-218, 2005.