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Transient Analysis of Laminated Composite Plates using Isogeometric Analysis

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Abstract

In this study, a NURBS-based isogeometric approach is developed for transient analysis of laminated composite plates using the classical laminated plate theory (CLPT). Based on the isoparametric concept, the NURBS basis function is employed for both the parameterization of the geometry and the approximation of the plate deflection. An efficient and easy to implement technique is used for imposition of essential boundary conditions by simply fixing the deflection of the first row or first two rows of control points from the desired boundary for simply supported or clamped boundaries, respectively. Some numerical examples for transient response analysis of laminated plates with various boundary conditions under dynamic loadings are considered. The numerical results are obtained and compared with other available solutions in the literature. The comparisons demonstrate the efficiency and accuracy of the proposed approach for such problems.

Keywords: transient analysis, laminated composite plates, classical laminated plate theory, isogeometric analysis, NURBS, Newmark integration.

1 Introduction

The complicated statement of most engineering problems and limitations of the analytical approaches have constantly motivated researchers to develop new numerical tools. FEM [1,2], BEM [3,4], smoothed FEM [5-11], and meshfree methods [12-16] are among various numerical methods which have been developed to accurately solve general engineering problems. Recently, Hughes and his co-workers have proposed a robustly computational isogeometric analysis (IGA) [17], which aims to unify the computer aided design (CAD) and the computer aided engineering (CAE) to benefit from the geometrical exactness operation in engineering problems. Following this approach, the CAD-shape functions, commonly the non-uniform rational B-splines (NURBS) are substituted for the Lagrange polynomial based shape functions in the CAE. The isoparametric concept is utilized where the NURBS basis functions are applied to both the geometric description and the solution field approximations, thereby suppressing the need for mesh generation and regeneration (as the design evolves) and, perhaps more importantly, permitting an exact representation of the complex geometry using just few elements. Nevertheless, numerous other main superiorities compared with the conventional finite element method can be pointed out as follows: (1) The computational cost is decreased significantly as the meshes are generated within the CAD in CAE; (2) Isogeometric analysis gives higher accurate results because of the smoothness and the higher order continuity between elements; (3) Mesh refinement is simple by re-indexing the parametric space without any interaction with the CAD system. Those characteristics of the IGA method have attracted many interests in a wide range of research areas such as structural vibrations, shells, incompressibility, fluid-structure interaction, turbulence, phase fields, contact, fracture and optimization.

In the present work, an isogeometric finite element method based on NURBS basis functions is developed for transient analysis of laminated composite plates using the classical laminated plate theory (CLPT). The finite element formulation based on the classical laminated plate theory requires elements with at least C^1-inter-element continuity. It is quite difficult to achieve such elements for free-form geometries when using the standard Lagrangian polynomials as basis functions. However, in the isogeometric analysis, higher order NURBS basis functions with an increased inter-element continuity can be easily obtained. Therefore, NURBS is well suited for the CLPT elements. The parameterization of the geometry and the approximation of the solution space for the deflection of plate are utilized using the NURBS basis functions. The governing equations of the laminated composite plate are transformed into a standard weak-form, which is then discretized into an isogeometric system of time-dependent equations to be solved by the unconditionally stable Newmark time integration scheme. The essential boundary conditions are enforced by a direct approach, and in this method, the clamped boundary condition is simply imposed by fixing the z-component of the first two rows of the control points from the desired boundary while the simply supported one is enforced by fixing the z-component of the first row of the control points. Some numerical examples of the laminated composite plates with different boundary conditions, fiber orientations and lay-up number are presented, discussed and compared with the analytical and other reference numerical solutions. As a consequence, it shows that the results derived from the isogeometric analysis are efficient and accurate for transient response of the laminated composite plates and also in a very good agreement with those of other conventional methods.

This paper is organized as follows. We briefly review the NURBS basis functions in the next section. In Section 3, the isogeometric formulations for forced vibration of laminated composite plates are briefly introduced due to the sake of completeness. Comparative studies and numerical applications for the transient analysis of laminated plates are presented in Section 4. We shall end with conclusions drawn from this work in the last section.

2 NURBS basis functions

In this section, a brief review of some technical features of B-spline and NURBS basis functions is presented. For more details on NURBS, the reader is referred to [18].

A non-uniform rational B-spline (NURBS) curve $\mathbf{C}(\xi)$ of order p is defined as

$$\mathbf{C}(\xi) = \sum_{i=1}^{n} R_{i,p}(\xi)\mathbf{P}_i \tag{1}$$

$$R_{i,p}(\xi) = \frac{N_{i,p}(\xi) w_i}{\sum_{j=1}^{n} N_{j,p}(\xi) w_j} \tag{2}$$

where $R_{i,p}$ stands for the univariate NURBS basis functions, $\mathbf{P}_i = (x_i, y_i);\, i = 1,2,...,n$ are a set of n control points, w_i are a set of n weights corresponding to the control points that must be non-negative and $N_{i,p}$ represents the B-spline basis function of order p. To construct a set of n B-spline basis functions of order p, a knot vector Ξ is defined in a parametric space as follows:

$$\begin{aligned} &\Xi = \left\{\xi_1, \xi_2, ..., \xi_{n+p+1}\right\} \quad \xi_i \le \xi_{i+1}, \\ &i = 1,2,...,n+p \end{aligned} \tag{3}$$

The parametric space is assumed to be $\xi \in [0,1]$. The knot vector is said to be open if the knots are repeated $p+1$ times at the start and end of its vector. In this study, only open knot vectors are considered. Given a knot vector, the univariate B-spline basis function $N_{i,p}$ can be constructed by the following Cox-de Boor recursion formula [18]

$$N_{i,0}(\xi) = \begin{cases} 1 & \text{if } \xi_i \le \xi \le \xi_{i+1} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

and

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi), \quad p = 1,\, 2,\, 3,\, \ldots\, . \tag{5}$$

The B-spline basis functions which are constructed from the open knot vectors have the interpolation feature at the ends of the parametric space.

Generally, a NURBS surface $\mathbf{S}(\xi,\eta)$ of order p in ξ direction and order q in η direction can be expressed as

$$\mathbf{S}(\xi,\eta)=\sum_{i=1}^{n}\sum_{j=1}^{m}R_{i,j}^{p,q}(\xi,\eta)\mathbf{P}_{i,j}=\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{N_{i,p}(\xi)M_{j,q}(\eta)w_{i,j}}{\sum_{\hat{i}=1}^{n}\sum_{\hat{j}=1}^{m}N_{\hat{i},p}(\xi)M_{\hat{j},q}(\eta)w_{\hat{i},\hat{j}}}\mathbf{P}_{i,j} \tag{6}$$

$$0\le\xi,\eta\le 1$$

where $R_{i,j}^{p,q}$ stand for the bivariate NURBS basis functions, $\mathbf{P}_{i,j}$ is a control mesh of $n\times m$ control points, $w_{i,j}$ are the corresponding weights, while $N_{i,p}$ and $M_{j,q}$ are the B-spline basis functions defined on the Ξ and H knot vectors, respectively.
The first derivative of $R_{i,j}^{p,q}(\xi,\eta)$ with respect to each parametric variable, e.g. ξ, is derived by simply applying the quotient rule to Equation (6) as

$$\frac{\partial R_{i,j}^{p,q}(\xi,\eta)}{\partial\xi}=\frac{\dfrac{\partial N_{i,p}(\xi)}{\partial\xi}M_{j,q}(\eta)w_{i,j}W(\xi,\eta)-\dfrac{\partial W(\xi,\eta)}{\partial\xi}N_{i,p}(\xi)M_{j,q}(\eta)w_{i,j}}{(W(\xi,\eta))^2} \tag{7}$$

and

$$W(\xi,\eta)=\sum_{\hat{i}=1}^{n}\sum_{\hat{j}=1}^{m}N_{\hat{i},p}(\xi)M_{\hat{j},q}(\eta)w_{\hat{i},\hat{j}} \tag{8}$$

$$\frac{\partial W(\xi,\eta)}{\partial\xi}=\sum_{\hat{i}=1}^{n}\sum_{\hat{j}=1}^{m}\frac{\partial N_{\hat{i},p}(\xi)}{\partial\xi}M_{\hat{j},q}(\eta)w_{\hat{i},\hat{j}} \tag{9}$$

Any higher order derivatives of the NURBS basis functions can be obtained in a similar fashion. The important properties of the NURBS basis functions can also be summarized as follows:

(1) Partition of unity, $\forall\xi,\ \sum_{i=1}^{n}R_{i,p}(\xi)=1$

(2) Non-negativity, $\forall\xi,\ R_{i,p}(\xi)\ge 0$

(3) Basis functions of order p are $p-m_i$ times continuously differentiable over a knot ξ_i, where m_i is the multiplicity of the value of ξ_i in the knot vector.

(4) Local support, i.e. the support of $R_{i,p}(\xi)$ is compact and contained in the interval $\left[\xi_i,\xi_{i+p+1}\right]$. In the two dimensional case, for a specified knot span (element), there are only $(p+1)\times(q+1)$ number of non-zero basis functions. Therefore, the total number of control points per element is $n_{en}=(p+1)\times(q+1)$.

It is worthwhile to note that in isogeometric analysis, by using the isoparametric concept, the NURBS basis is employed for both the parameterization of the geometry and the approximation of the solution field, which is the plate deflection $w(\mathbf{x})$ in this paper, as follows:

$$w^h(\mathbf{x}(\boldsymbol{\xi})) = \sum_{I=1}^{n\times m} \phi_I(\boldsymbol{\xi}) w_I \tag{10}$$

$$\mathbf{x}(\boldsymbol{\xi}) = \sum_{I=1}^{n\times m} \phi_I(\boldsymbol{\xi}) \tilde{\mathbf{x}}_I \tag{11}$$

In all the above equations, $\boldsymbol{\xi} = (\xi, \eta)$ is the parametric coordinates, $\mathbf{x} = (x, y)$ is the physical coordinates, $\tilde{\mathbf{x}}_I$ represents the control points of a $n \times m$ control mesh, w_I represents the deflection of the plate at each control point, and $\phi_I(\boldsymbol{\xi})$ are the bivariate NURBS basis functions of order p and q in ξ and η directions, respectively.

3 Governing equations and discretization

3.1 Forced vibration analysis

Let us consider a symmetrically laminated composite plate as depicted in Figure 1 under Cartesian coordinate system with the thickness h in the $z-$direction and the fiber orientation θ of a layer. The displacements of the plate in the $x-$, $y-$, $z-$directions are denoted as u, v, w, respectively.

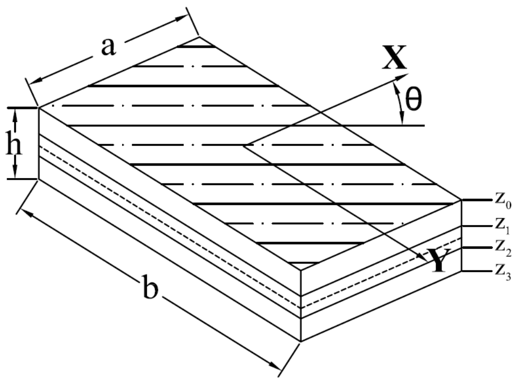


Figure 1: A schematic composite laminated plate

According to classical laminated plate theory [19], the displacement fields of the plate are given by,

$$\mathbf{u}=\{u \quad v \quad w\}^T=\left\{-z\frac{\partial}{\partial x} \quad -z\frac{\partial}{\partial y} \quad 1\right\}^T w=\hat{\mathbf{L}}w \tag{12}$$

It's worth mentioning that following the classical laminated plate assumption [19], only the deflection of the plate $w(\mathbf{x})$ is chosen as the independent variable and approximated by the NURBS basis functions while the other two displacement components $u(\mathbf{x})$ and $v(\mathbf{x})$ can be directly obtained from the deflection $w(\mathbf{x})$ through Equation (12).

The constitutive equation describing the relationship between the strains and stresses can be expressed as,

$$\boldsymbol{\sigma}_p=\mathbf{D}\boldsymbol{\varepsilon}_p \tag{13}$$

where $\boldsymbol{\varepsilon}_p$ and $\boldsymbol{\sigma}_p$ are pseudo-strains and –stresses, respectively. Due to the assumption of classical laminated plate, $\mathbf{D}$ can be written as,

$$\mathbf{D}=\begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \tag{14}$$

$$D_{IJ}=\frac{1}{3}\sum_{k=1}^{N}(\bar{Q}_{IJ})_k(z_k^3-z_{k-1}^3), \quad I,J=1,2,6 \tag{15}$$

In the above equation, N is the number of layers of the composite laminated plate and $\bar{Q}_{IJ}$ is defined based on the material properties and the fiber orientation θ of each layer [19].

The pseudo-strains and pseudo-stresses of the plate are denoted as,

$$\boldsymbol{\varepsilon}_p=\left\{-\frac{\partial^2}{\partial x^2} \quad -\frac{\partial^2}{\partial y^2} \quad -2\frac{\partial^2}{\partial x\partial y}\right\}^T w=\mathbf{L}w \tag{16}$$

$$\boldsymbol{\sigma}_p=\{M_x \quad M_y \quad M_{xy}\}^T \tag{17}$$

In which M_x, M_y and M_{xy} are bending and twisting moments, respectively.

For the forced vibration analysis, the weak form of the elastodynamic undamped equilibrium equation of the plate can be expressed as,

$$\begin{aligned}\frac{d}{dt}\int_\Omega \rho\frac{\partial}{\partial \dot{w}}(\hat{\mathbf{L}}\dot{w})^T(\hat{\mathbf{L}}\dot{w})d\Omega+\int_A\frac{\partial}{\partial w}(\mathbf{L}w)^T\mathbf{D}(\mathbf{L}w)dA=\int_{\Gamma_t}\frac{\partial}{\partial w}(\hat{\mathbf{L}}w)^T\bar{\mathbf{t}}d\Gamma \\ +\int_\Omega\frac{\partial}{\partial w}(\hat{\mathbf{L}}w)^T\mathbf{b}d\Omega\end{aligned} \tag{18}$$

where Ω stands for the volume of the plate, $\bar{\mathbf{t}}$ and $\mathbf{b}$ are the prescribed boundary forces and the body force vector, respectively, A stands for the area of the plate, Γ_t is the surface of the plate edge in which the prescribed boundary forces are applied, and ρ is the mass density of the material. In the above equation, the over-dot denotes the differentiation with respect to time. By substituting the deflection w from Equation (10) into the weak form defined as Equation (18), the undamped dynamic discrete equations can be derived as,

$$\mathbf{M}\ddot{\mathbf{w}} + \mathbf{K}\mathbf{w} = \mathbf{f} \tag{19}$$

where $\mathbf{w}$ and $\ddot{\mathbf{w}}$ are the vectors of the deflection and acceleration at the control points. $\mathbf{K}$ and $\mathbf{M}$, respectively, stand for global stiffness and mass matrices which are defined as,

$$K_{IJ} = \int_A \mathbf{B}_I^T \mathbf{D} \mathbf{B}_J dA \tag{20}$$

$$M_{IJ} = \int_A (I_m \frac{\partial \phi_I}{\partial x} \frac{\partial \phi_J}{\partial x} + I_m \frac{\partial \phi_I}{\partial y} \frac{\partial \phi_J}{\partial y} + \rho h \phi_I \phi_J) dA \tag{21}$$

$$\mathbf{B}_I = \left\{ -\frac{\partial^2 \phi_I}{\partial x^2} \quad -\frac{\partial^2 \phi_I}{\partial y^2} \quad -2\frac{\partial^2 \phi_I}{\partial x \partial y} \right\}^T \tag{22}$$

in which $I_m = \rho h^3 / 12$ is the mass moment of inertia. $\mathbf{f}$ is the global force vector which is defined as,

$$f_I = \int_\Omega \tilde{\mathbf{B}}_I^T \mathbf{b} d\Omega + \int_{\Gamma_t} \tilde{\mathbf{B}}_I^T \bar{\mathbf{t}} d\Gamma \tag{23}$$

$$\tilde{\mathbf{B}}_I = \left\{ -z \frac{\partial \phi_I}{\partial x} \quad -z \frac{\partial \phi_I}{\partial y} \quad \phi_I \right\}^T \tag{24}$$

Because the plate considered here is only subjected to transverse load (in the z direction), the body force vector has the following form,

$$\mathbf{b} = \{0 \quad 0 \quad b_z\}^T \tag{25}$$

Therefore, the global force vector $\mathbf{f}$ can be rewritten as,

$$f_I = \int_\Omega \phi_I b_z d\Omega + \int_{\Gamma_t} \tilde{\mathbf{B}}_I^T \bar{\mathbf{t}} d\Gamma = \int_A \phi_I q_z dA + \int_{\Gamma_t} \tilde{\mathbf{B}}_I^T \bar{\mathbf{t}} d\Gamma \tag{26}$$

where q_z is the force per unit area.

It should be noted that for forced vibration analysis, the approximation function defined in Equation (10), is a function of both space and time. For the displacements and accelerations at time $t + \Delta t$, the dynamic equations presented in Equation (19) should also be considered at time $t + \Delta t$ as follows,

$$\mathbf{M}\ddot{\mathbf{w}}_{t+\Delta t} + \mathbf{K}\mathbf{w}_{t+\Delta t} = \mathbf{f}_{t+\Delta t} \tag{27}$$

To solve this second order time dependent problem, several methods have been proposed such as, Wilson, Newmark, Houbolt, Crank–Nicholson, etc. [20,21]. In this paper, Equation (27) is solved by the Newmark time integration method. The formulation of this method can be given in the following form [20,22],

$$\ddot{\mathbf{w}}_{t+\Delta t} = \frac{1}{\alpha(\Delta t)^2}(\mathbf{w}_{t+\Delta t} - \mathbf{w}_t) - \frac{1}{\alpha\Delta t}\dot{\mathbf{w}}_t - \left(\frac{1}{2\alpha} - 1\right)\ddot{\mathbf{w}}_t \tag{28}$$

$$\dot{\mathbf{w}}_{t+\Delta t} = \dot{\mathbf{w}}_t + \left[(1-\delta)\ddot{\mathbf{w}}_t + \delta\ddot{\mathbf{w}}_{t+\Delta t}\right]\Delta t \tag{29}$$

By substituting Equations (28) and (29) into Equation (27), the dynamic response at time $t + \Delta t$ can be obtained. Because the Newmark time integration method is an implicit method, the initial conditions of the state at $t = t_0$ are thus assumed to be known $(\mathbf{w}_0, \dot{\mathbf{w}}_0, \ddot{\mathbf{w}}_0)$ and the new state at the time $t_1 = t_0 + \Delta t$, $(\mathbf{w}_1, \dot{\mathbf{w}}_1, \ddot{\mathbf{w}}_1)$ is needed to be determined correspondingly. Moreover, the Newmark method is unconditionally stable if $\delta \geq 0.5$ and $\alpha \geq \frac{1}{4}(\delta + 0.5)^2$. Therefore, we choose $\delta = 0.5$ and $\alpha = 0.25$ in order to guarantee the unconditionally stability of the Newmark method.

3.2 Essential boundary conditions

In this paper, a simple direct technique has been used for imposition of essential boundary conditions in plate problems in particular for dealing with the fully clamped boundary condition which was firstly described by Kiendl et al. [23] for analysis of shell structures. Using this technique, clamped boundary condition can be simply imposed by fixing the z-component of the first two rows of control points from the desired boundary. It is based on the fact that the slopes at the boundary of a NURBS surface are determined by the first two rows of control points from this boundary [23]. Simply supported boundary condition is also imposed by fixing the z-component of the first row of control points from the boundary. It can be seen that this approach for imposing essential boundary conditions, in comparison with other available methods [24,25], is very easy to implement and computationally efficient.

4 Numerical examples

In order to demonstrate the accuracy and validity of the proposed approach for transient analysis of laminated composite plates, several numerical examples with different transient loadings and boundary conditions are investigated in this section. The obtained results of present isogeometric approach are verified by comparing with other numerical or analytical solutions available in the literature. For all calculations, cubic order NURBS basis function over an 11×11 control mesh is used. In the following examples, all layers of the laminated plate are assumed to have the same thicknesses and material properties. The time step $dt = 0.01\,\text{ms}$ is employed in all calculations, unless otherwise stated. For the convenience, the boundaries of the plate are denoted as: simply supported (S), fully clamped (C), and completely free (F) edges. For example, notation SCSC means the right and left boundaries are simply supported while the bottom and upper boundaries are clamped.

4.1 Example 1

A fully simply supported three-layer square laminated plate arranged as $(0°, 90°, 0°)$ is considered. This example was also studied by Wang et al. [26] and is chosen here to demonstrate the accuracy of the IGA in dynamic analysis of plates under different transient loads including step, triangular, sine and explosive blast loads. The material properties for this example are given as follows,

$$E_1 = 172.369\,\text{GPa},\quad E_2 = 6.895\,\text{GPa},\quad G_{12} = 3.448\,\text{GPa},\quad \nu_{12} = 0.25,\quad \rho = 1603.03\,\text{kg/m}^3$$

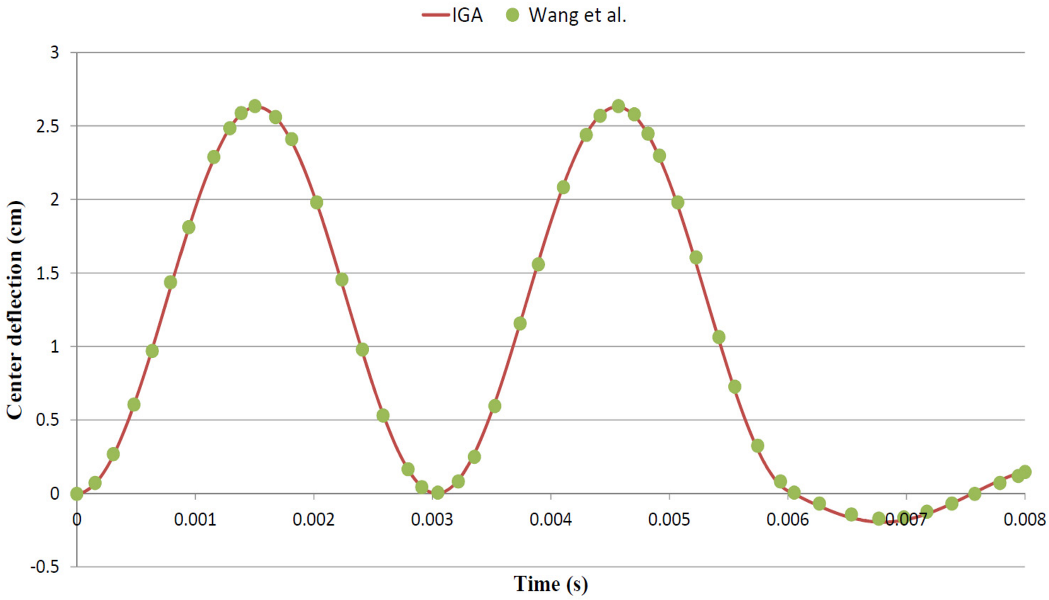
The plate length is assumed to be $a = b = 20h$ in which $h = 0.0381\,\text{m}$ is the total thickness of the plate. The plate is subjected to a transverse load which is sinusoidally distributed in spatial domain and varies with time as,

$$q(x, y, t) = q_0 \sin(\frac{\pi x}{a}) \sin(\frac{\pi y}{b}) F(t) \tag{30}$$

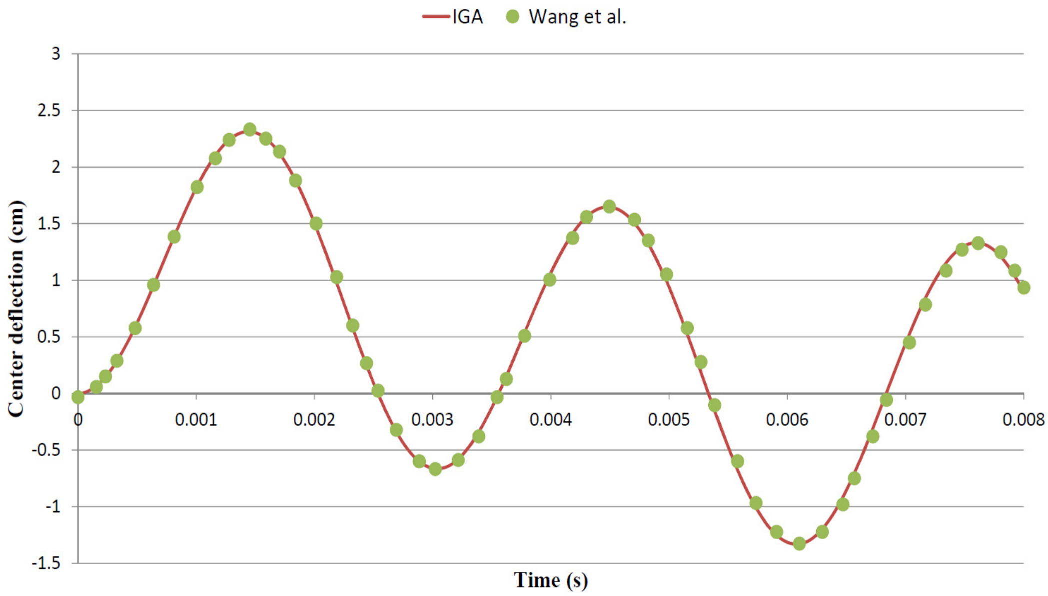
$$F(t) = \begin{cases} \begin{Bmatrix} 1 & 0 \le t \le t_1 \\ 0 & t > t_1 \end{Bmatrix} & \textit{step loading} \\ \begin{Bmatrix} 1 - t/t_1 & 0 \le t \le t_1 \\ 0 & t > t_1 \end{Bmatrix} & \textit{triangular loading} \\ \begin{Bmatrix} \sin(\pi t/t_1) & 0 \le t \le t_1 \\ 0 & t > t_1 \end{Bmatrix} & \textit{sine loading} \\ e^{-\gamma t} & \textit{explosive blast loading} \end{cases} \tag{31}$$

where $t_1 = 0.006\,\text{s}$, $\gamma = 330\,\text{s}^{-1}$ and $q_0 = 3.448\,\text{MPa}$.

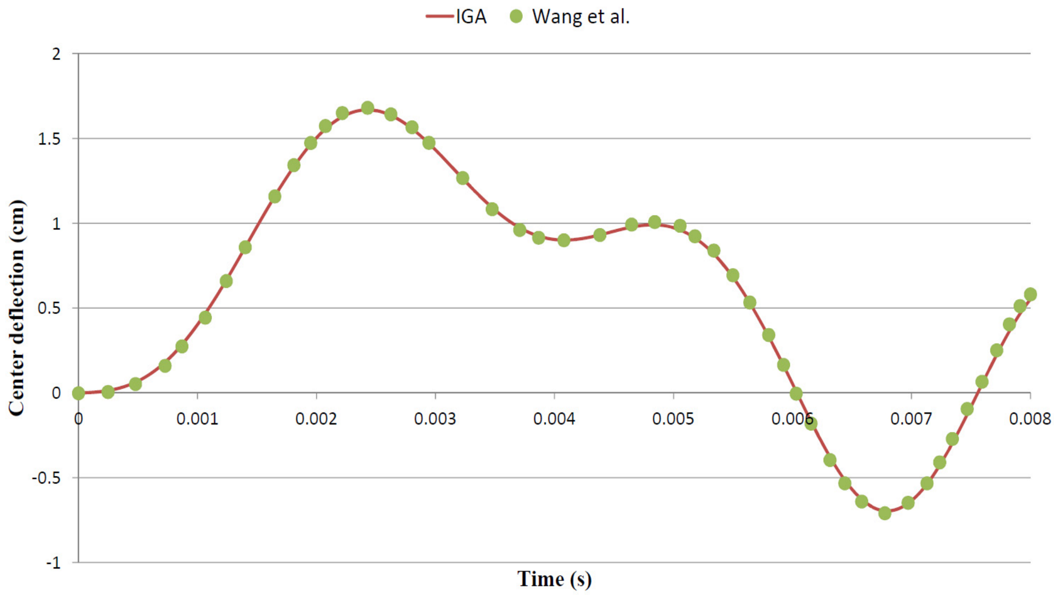
Figure 2 shows the time histories of central deflection of the plate under various dynamic loadings. The results of isogeometric analysis are compared with those obtained by Wang et al. [26] using the strip element method (SEM). As expected, it can be observed in Figure 2 that the IGA results match well with the reference solutions.



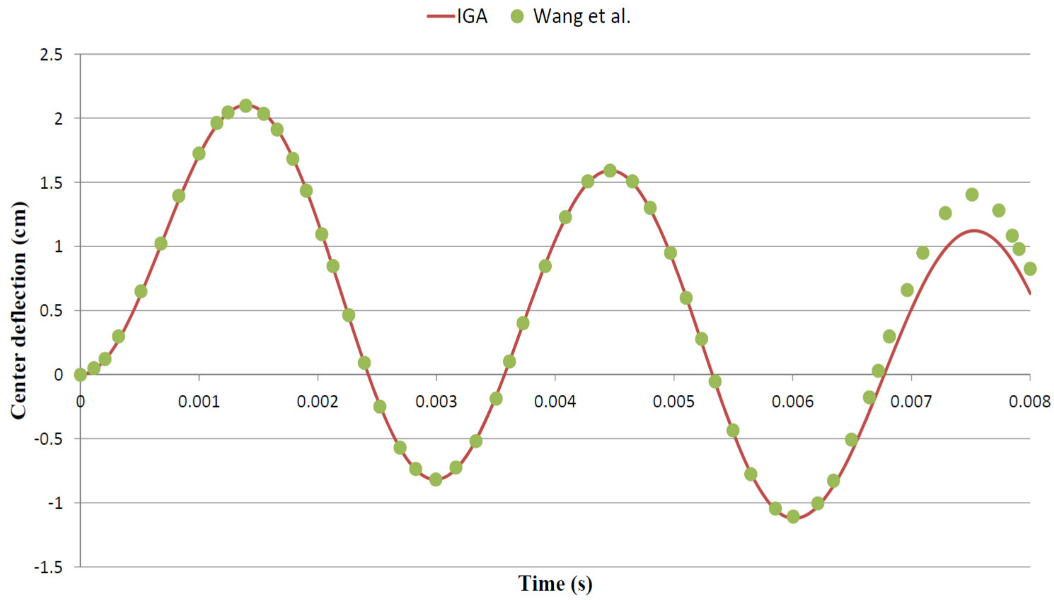
(a) step loading



(b) triangular loading



(c) sine loading



(d) explosive blast loading

Figure 2: Central deflection versus time for a $(0°, 90°, 0°)$ square laminated plate subjected to various dynamic loadings (a) step loading (b) triangular loading (c) sine loading (d) explosive blast loading.

4.2 Example 2

In this example, a four layer square laminated plate arranged as $(30°, -30°, -30°, 30°)$ is considered. The following geometrical and material properties are used in this example:

$$a = b = 1.27\,\mathrm{m}, \quad h = 0.0254\,\mathrm{m}$$

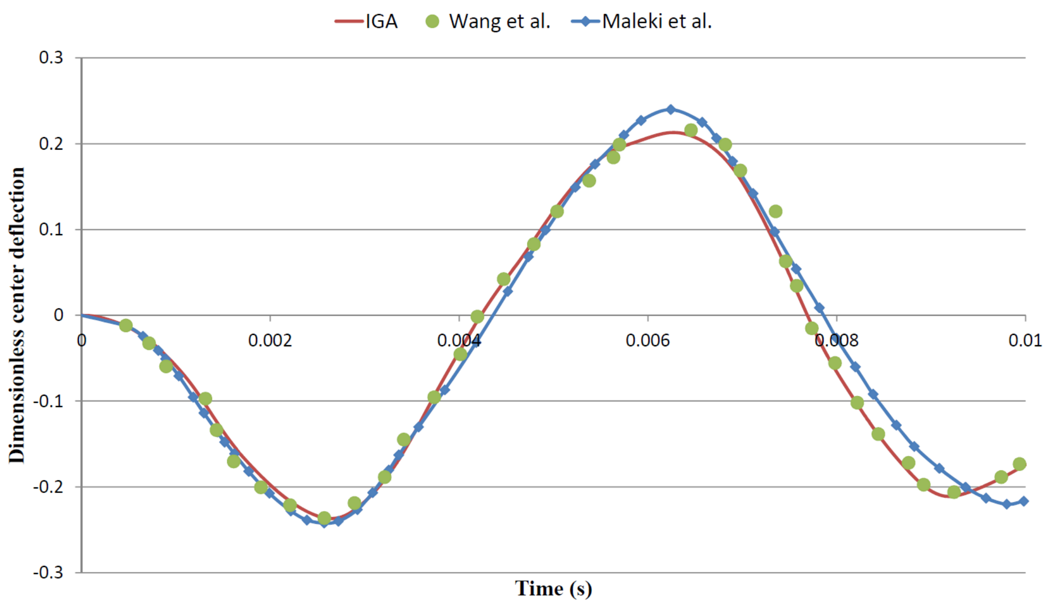
$$E_1 = 131.69\,\mathrm{GPa}, \quad E_2 = 8.55\,\mathrm{GPa}, \quad G_{12} = 6.67\,\mathrm{GPa}, \quad \nu_{12} = 0.3, \quad \rho = 1610\,\mathrm{kg/m^3}$$

A conventional blast load (combination of triangular and blast loads) is applied to the plate, while a uniform pressure distributed in spatial domain. This loading function can be defined as,

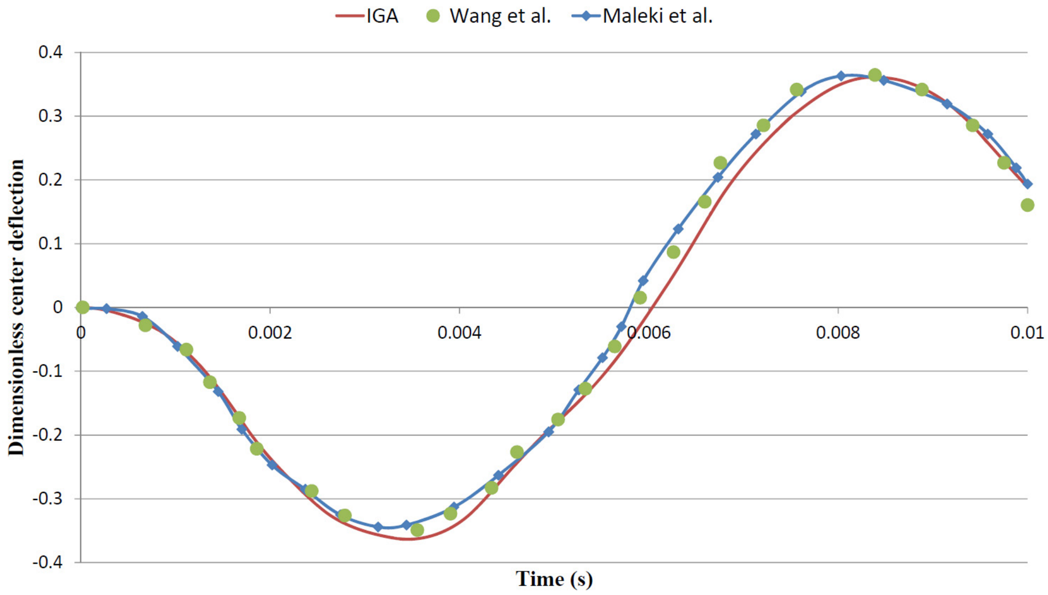
$$q(x,y,t) = q_0\left(1 - \frac{t}{t_2}\right)e^{-\alpha_1 t/t_2} \tag{32}$$

where $q_0 = 68.95\,\mathrm{kPa}$,$t_2 = 0.004\,\mathrm{s}$ and $\alpha_1 = 1.98$.

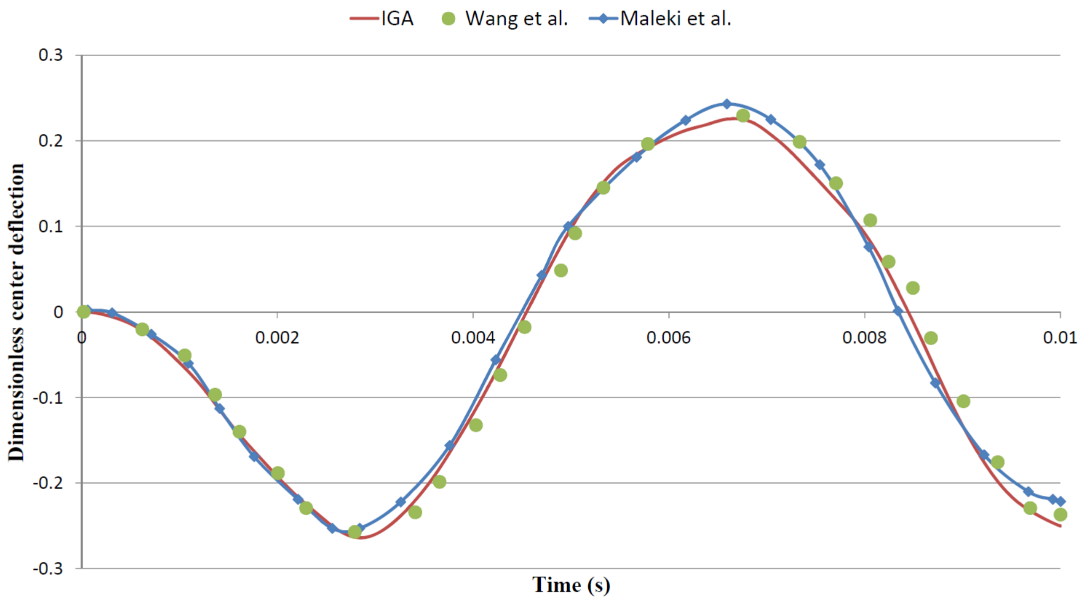
For this problem, the dimensionless deflection parameter $\bar{w} = 100w\left(E_2 h^3 / q_0 a^4\right)$ is used. Figure 3 shows dimensionless central deflection versus time for the plate with different boundary conditions. For comparison purpose, the results obtained by Wang et al. [26] using SEM method and those obtained by Maleki et al. [27] using generalized differential quadrature method (GDQ) are also presented altogether. Similarly, a good agreement among approaches for all types of boundary conditions is obtained.



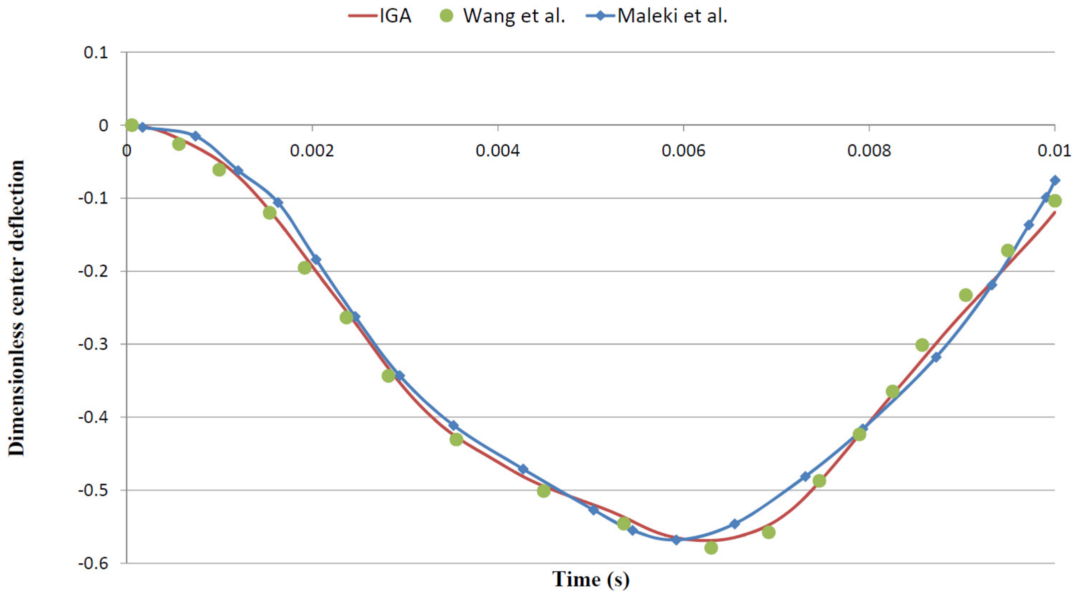
(a) CCCC


IGA
Wang et al.
Maleki et al.
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0
0.002
0.004
0.006
0.008
0.01
Dimensionless center deflection
Time (s)

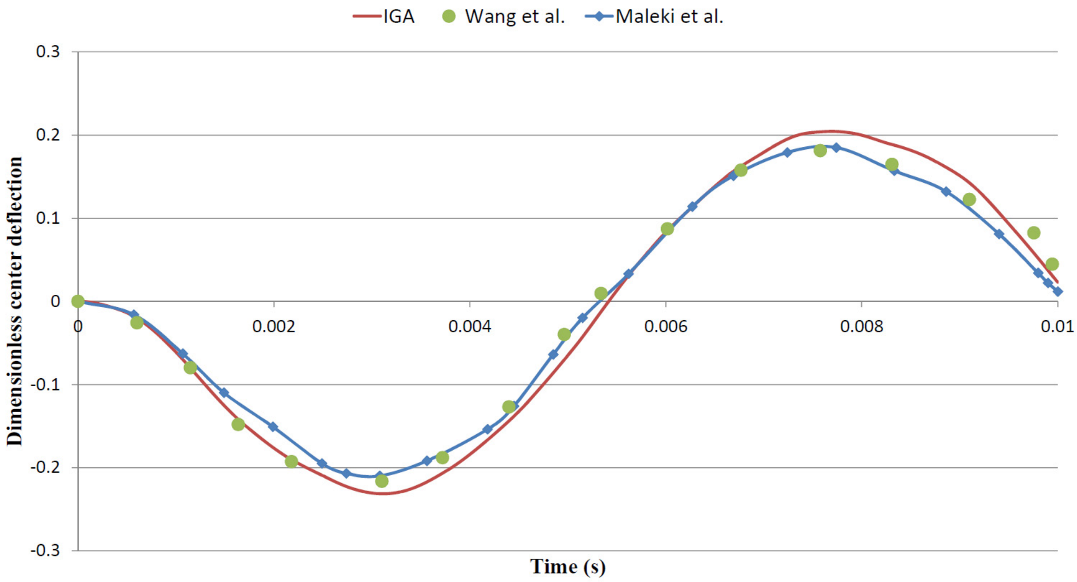
(b) SCSC


IGA
Wang et al.
Maleki et al.
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0
0.002
0.004
0.006
0.008
0.01
Dimensionless center deflection
Time (s)

(c) CSCS



(d) FCFC



(e) CFCF

Figure 3: Dimensionless central deflection versus time for a $(30°, -30°, -30°, 30°)$ square laminated plate subjected to conventional blast load with different boundary conditions (a) CCCC (b) SCSC (c) CSCS (d) FCFC (e) CFCF.

5 Conclusion

In this paper, we have successfully presented an application of the NURBS-based isogeometric finite element approach to transient analysis of thin laminated composite plates. The parameterization of the geometry and the approximation of the solution space for the deflection of plate are utilized using the NURBS basis functions. The governing equations of the laminated composite plate are transformed into a standard weak-form. It is then discretized into an isogeometric system of time-dependent equations to be solved by the unconditionally stable Newmark time integration scheme. The essential boundary conditions are enforced by a direct approach. Some numerical examples of the laminated composite plates under various dynamic loadings with different boundary conditions, fibre orientations and lay-up number are presented, discussed and compared with the analytical and other reference numerical solutions. From the gained numerical results, it definitely shows that the proposed approach can yield accurate solutions for the transient analysis of laminated composite plates.

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